b. If the base function $y = x^2$ is considered, only the modifications of parameters **b** and **c** would generate a change in the position of the vertex.

c. When parameter **b** varies in the equation $y = x^2 + bx$, the vertex moves along the parabola whose equation is $y = -x^2$.

Practice 4.1 — Page 207

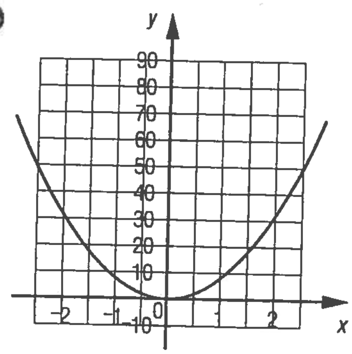
1.

	①	②	③
a)	$x = 0$	$x = \frac{5}{2}$	$x = -\frac{1}{2}$
b)	(0, 0)	$\left(\frac{5}{2}, -\frac{1}{4}\right)$	$\left(-\frac{1}{2}, -\frac{243}{4}\right)$

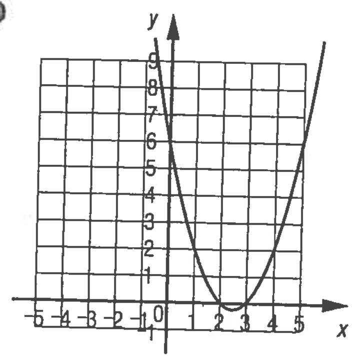
	④	⑤	⑥
a)	$x = 0$	$x = 1$	$x = -\frac{17}{20}$
b)	(0, 16)	$\left(1, \frac{5}{2}\right)$	$\left(-\frac{17}{20}, -\frac{2209}{200}\right)$

c) *Several possible answers.*

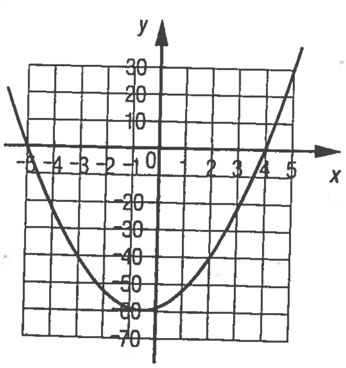
d) ①



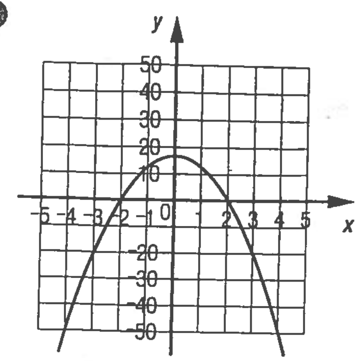
②



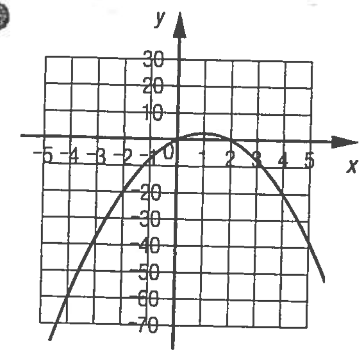
③



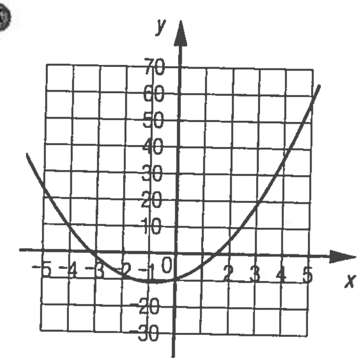
④



⑤



⑥



2.

	①	②
a)	(3, -1)	(-1, 0)
b)	2 and 4	-1
c)	$f_1(x) = x^2 - 6x + 8$	$f_2(x) = -2x^2 - 4x - 2$
d)	$h = \frac{6}{2} = 3$ $k = \frac{4 \cdot 1 \cdot 8 - 6^2}{4} = -1$	$h = \frac{4}{-4} = -1$ $k = \frac{4 \cdot -2 \cdot -2 - 4^2}{-8} = 0$
e)	2 and 4	-1

	③	④
a)	(-2, 12)	(4, -6)
b)	-4 and 0	None
c)	$f_3(x) = -3x^2 - 12x$	$f_4(x) = -2x^2 + 16x - 38$
d)	$h = \frac{12}{-6} = -2$ $k = \frac{4 \cdot -3 \cdot 0 - 12^2}{-12} = 12$	$h = \frac{-16}{-4} = 4$ $k = \frac{4 \cdot -2 \cdot -38 - 16^2}{-8} = -6$
e)	-4 and 0	None

3.

	1	2
a)	(5, -15)	(-1, -14)
b)	$f_1(x) = (x - 5)^2 - 15$	$f_2(x) = 2(x + 1)^2 - 14$

	3	4
a)	(2, 22)	$\left(-\frac{3}{4}, \frac{23}{8}\right)$
b)	$f_3(x) = -3(x - 2)^2 + 22$	$f_4(x) = 2\left(x + \frac{3}{4}\right)^2 + \frac{23}{8}$

4. If k = c, then b = 0 and h = 0.
In fact, $k = \frac{4ac - b^2}{4a} = c - \frac{b^2}{4a}$. If k = c, then $\frac{b^2}{4a} = 0$, therefore b = 0. Since $h = \frac{-b}{2a}$, if b = 0, therefore h = 0.

Practice 4.1 *(cont'd)* Page 208

5. **a)** 2 zeros, -6 and 1. **b)** 1 zero, -2.
c) 2 zeros, -6 and -4. **d)** No zeros.
e) 2 zeros, $-2\sqrt{2}$ and $-\sqrt{2}$. **f)** 1 zero, 1.

6.

	1	2	3
a)	$x = 3$	$x = -2$	$x = \frac{1}{6}$
b)	[-4, +∞[	]-∞, 0]	$\left]-\infty, -\frac{47}{12}\right]$
c)	(3, -4)	(-2, 0)	$\left(\frac{1}{6}, -\frac{47}{12}\right)$
d)	5	-16	-4
e)	Minimum of -4	Maximum of 0	Maximum of $-\frac{47}{12}$
f)	1 and 5	-2	None
g)	[3, +∞[	]-∞, -2]	$\left]-\infty, \frac{1}{6}\right]$
h)	]-∞, 3]	[-2, +∞[	$\left[\frac{1}{6}, +\infty\right[$
i)	]-∞, 1] ∪ [5, +∞[	{-2}	None
j)	[1, 5]	ℝ	ℝ

7. **a) 1)** After 1,5 s. **2)** After approximately 6.6 s.
b) After 1 s or 2 s.

8. **a) 1)** $f_1(x) = (x - 2)^2 - 3$
2) $f_2(x) = \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$
3) $f_3(x) = \left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$
b) *Several ways to proceed. Example:*
$f_4(x) = 3(x^2 - 2x) + 5$
$= 3(x^2 - 2x + 1) - 3(1) + 5$
$= 3(x - 1)^2 + 2$

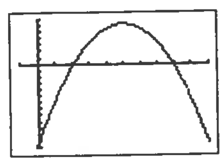
Practice 4.1 *(cont'd)* Page 209

9. **a)** The maximum temperature was 1.31°C and was reached around 8:43 p.m.
b) Around 1:51 p.m.
c) The *y*-intercept is -10.7. That is what the temperature would have been (in °C) at the beginning of the day on February 5, 2008 at midnight if the model could be applied to the whole day.

10. **a)** $R(x) = -0.8x^2 + 32x$
b) Domain: [0, 40] Range: [0, 320]
c) The zeros are 0 and 40. They represent the two prices that would allow for a profit of zero.
d) \$20
e) 1) \$10.65 **2)** \$29.35

Practice 4.1 *(cont'd)* Page 210

11. **a) 1)** 4.9 m **2)** 19.6 m
b) After approximately 3.3 s.
c) After approximately 3.0.
d) a) 1) 4.9 m **2)** 19.6 m
b) After approximately 4.7 s.
c) After approximately 4.5 s.

12. **a)** *Several possible answers. Example:*
Xmin=-5
Xmax=50
Ymin=-130
Ymax=70
b)

Vertex: (25, 62.5).
y-intercept: -125.
x-intercept: approximately 10.56 and 39.44.

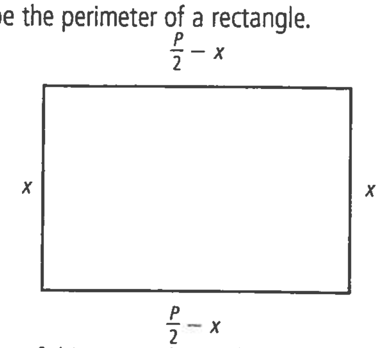
Practice 4.1 *(cont'd)* Page 211

13. **a)** $f(x) = 2x^2 - 120x + 3600$
b) 3600. This value represents the area of square ABCD (in cm^2). When distance *x* is worth 0, the two squares coincide.
c) 1800. This value represents the minimal area of square EFGH (in cm^2) which can be drawn on the inside of square ABCD.
d) Function f is increasing in [30, 60].
Function f is decreasing in [0, 30].

14. **a)** $f(x) = -2x^2 + 10x$
b)]0, 5[
c)]0, 12.5]
d) For $x = 2.5$.

e) There are two solutions:
The two parallel sides can measure $\frac{5-\sqrt{5}}{2}$ m (approximately 1.38 m) and the other side should therefore measure $(5+\sqrt{5})$ m (approximately 7.24 m).
or
The two parallel sides measure $\frac{5+\sqrt{5}}{2}$ m (approximately 3.62 m) and the other side will therefore measure $(5-\sqrt{5})$ m (approximately 2.76 m).

15. Let P be the perimeter of a rectangle.



The area of this rectangle can be represented by the function $f(x) = x\left(\frac{P}{2} - x\right)$ or, in the general form, by $f(x) = -x^2 + \frac{P}{2}x$.
With parameter **a** being negative, the function allows a maximum that is obtained once the value of x assumes the first coordinate of the vertex.

The maximum is therefore reached when
$x = \frac{-b}{2a} = \frac{-\left(\frac{P}{2}\right)}{2(-1)} = \frac{P}{4}$.
In this case, the rectangle has 4 congruent sides with lengths of $\frac{P}{4}$.
Among all the rectangles with a given perimeter, the square has the largest area.

SECTION 4.2

The quadratic function and inequalities

Problem

Page 212

Calculating to the nearest metre, the radius of O'Neill's imagined cylinder must have a radius of at least 2.453 km.

Several possible approaches. Example:

Since the height of the cylinder is 30 km, its surface area is a function of its radius r, that is $A(r) = 2\pi r^2 + 60\pi r$.

If context is not considered, the interval within which the function is increasing is $[-30, +\infty[$. Since the value of r, according to the context, is greater than 0, one can affirm that this function is increasing for the whole domain of the definition of the variable.

The minimum radius required for the cylinder to have an area of 500 km² is therefore the positive solution of the equation $500 = 2\pi r^2 + 60\pi r$, which equals $2\pi r^2 + 60\pi r - 500 = 0$.

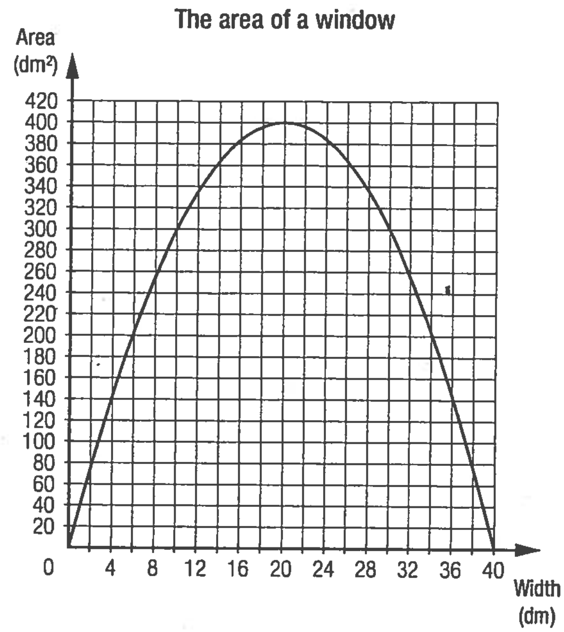
The solution gives $x = \frac{-60\pi + \sqrt{3600\pi^2 + 4000\pi}}{} = -15 + 5\sqrt{9 + \frac{10}{\pi}} \approx 2.4521$.

Activity 1

Page 213

a. $f(x) = -x^2 + 40x$

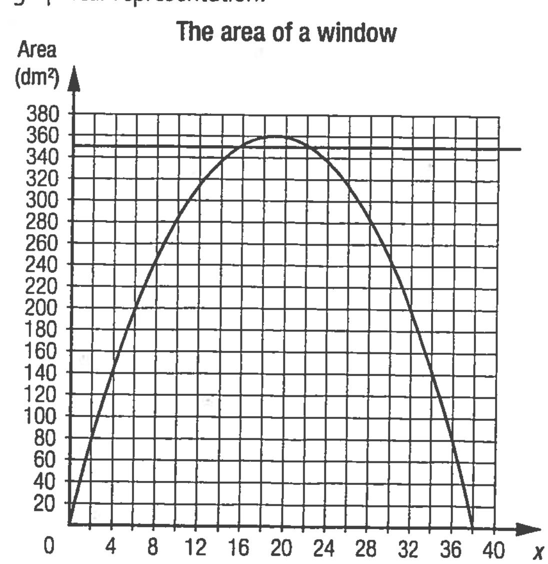
b.



c. $20 - \sqrt{50}$ or $20 + \sqrt{50}$.

d. Any value of x within the interval [12.93, 27.07].

e. *Several possible approaches. Example:*
The function is $g(x) = -x^2 + 38x$. The following is its graphical representation:



One can see that a part of the curve is above 350. It is therefore possible to build a window with an area that is larger than 350 dm² by using the window model.

Activity 1 (*cont'd*)

Page 214

a. $(4x-6)^2 < x^2 + (5x-10)^2$

$16x^2 - 48x + 36 < x^2 + 25x^2 - 100x + 100$

$16x^2 - 48x + 36 < 26x^2 - 100x + 100$

$-10x^2 + 52x - 64 < 0$

$5x^2 - 26x + 32 > 0$

b. $(x-2)(5x-16)$

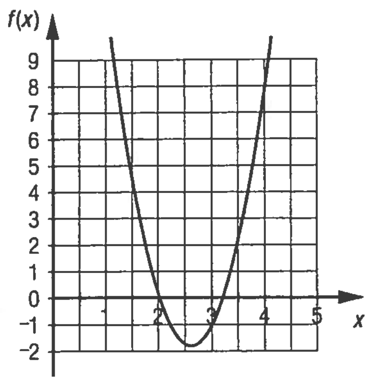
c. A and B have the same sign.

d.

Value of x	$]-\infty, 2[$	2	$]2, \frac{16}{5}[$	$\frac{16}{5}$	$]\frac{16}{5}, +\infty[$
Sign of $(x-2)$	−	0	+	+	+
Sign of $(5x-16)$	−	−	−	0	+

e. $]-\infty, 2[\cup]3{,}2, +\infty[$

f. By using the graph and zeros, the conclusion is the same as previously stated.

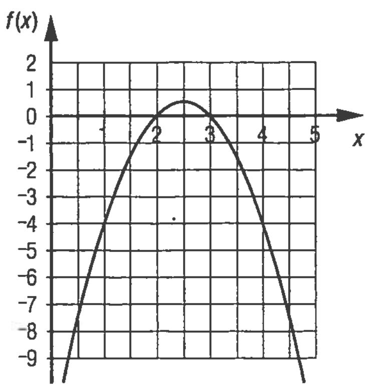


g. $]3{,}2, 4[$

Practice 4.2

Page 217

1. a)



b) $[2, 3]$

c) 1) $]1, 4[$ 2) $\left]-\infty, \frac{5-\sqrt{5}}{2}\right] \cup \left[\frac{5+\sqrt{5}}{2}, +\infty\right[$

3) $\frac{5}{2}$ 4) $\mathbb{R}$

2.

Value of x	$]-\infty, -2[$	-2	$]-2, \frac{5}{2}[$	$\frac{5}{2}$	$]\frac{5}{2}, +\infty[$
Sign of $(5-2x)$	+	+	+	0	−
Sign of $(x+2)$	−	0	+	+	+

The solution set is $]-\infty, -2] \cup \left[\frac{5}{2}, +\infty\right[$.

3. a) $]-\infty, -3[\cup \left]\frac{1}{2}, +\infty\right[$ b) $\left]\frac{1}{2}, 2\right[$

c) $\left[-\frac{5}{2}, \frac{7}{2}\right]$ d) $]-\infty, -8[\cup]8, +\infty[$

e) $]-\infty, 1[\cup]1, +\infty[$ f) $\left]-\frac{2}{5}, 2\right[$

g) $]-\infty, -3] \cup [-2, +\infty[$ h) $]-\infty, -7] \cup [1, +\infty[$

i) $]-\infty, -3[\cup]2, +\infty[$

4. a) $]-\infty, -3[\cup]-2, +\infty[$ b) $\varnothing$ c) $\mathbb{R}$

d) $\mathbb{R}$ e) $\left[0, \frac{2}{3}\right]$

f) $\left]-\infty, \frac{2-\sqrt{10}}{2}\right] \cup \left[\frac{2+\sqrt{10}}{2}, +\infty\right[$

5. a) People who are 22 years old and under and people who are 68 years old and over.

b) It is impossible to get this discount since the obtained inequality does not have a solution.

Practice 4.2 (*cont'd*)

Page 218

6. a) The area of the casing is $6c^2 + 80c$.
The inequality associated to this situation is $6c^2 + 80c \leq 600$.
Therefore, $6c^2 + 80c - 600 \leq 0$.

b) 1) Yes. 2) Yes. 3) No.

c) If one were to pinpoint a number to the nearest millimetre, the edge of the cube (in dm) should be included in the interval $]0, 5{,}35]$.

7. If one were to pinpoint a number to the nearest centimetre, this addition (in m) should be included in the interval $]0, 9{,}72]$.

Practice 4.2 (*cont'd*)

Page 219

8. a) $f(x) = 0.5x^2 + 0.5x$

b) 141 people would have to attend.

9. a) 1) 0 N 2) 370 N 3) 37 000 N

b) If one were to pinpoint a number to the nearest centimetre, the stretching of the elastic throughout the test should be found within an interval of 79 cm to 96 cm.
Several possible answers concerning the procedure to follow in order to complete the test. Example:
Joelle could use a set-up that measures 1.05 m in length to which she would attach an elastic band measuring 0.20 cm; this set-up would allow the elastic to stretch 85 cm about a thousand times.

10. Parameter a must be found in the interval $\left[\frac{20}{3}, 10\right]$.
Several possible answers. Example:
The inequality $ax^2 - 4x + 4 \geq 3.4$ is equal to $ax^2 - 4x + 0.6 \geq 0$.
The function $g(x) = ax^2 - 4x + 0.6$ is analyzed. With parameter a being positive, the parabola representing this function is opened upwards. This function would only be

positive for all values of x if it has only one zero or none (if the vertex is situated on or above the x-axis). In other words, the discriminant must be less than or equal to 0.
The discriminant:
$b^2 - 4ac = (-4)^2 - 4a(0.6) = 16 - 2.4a$.
The solution of the inequality $16 - 2.4a \leq 0$ gives $a \geq \frac{20}{3}$.

Practice 4.2 (*cont'd*) — Page 220

11. a) 3.05 kW

b) By taking into account the wind speeds to the nearest km/h, the wind must blow at a speed of 33 km/h to 55 km/h to provide enough energy.

12. a) Lateral area of the cone: $\pi r^2 + 80\pi r$
Area of the square: $(2r + 16)^2 = 4r^2 + 64r + 256$
Double the area of the square $= 8r^2 + 128r + 512$

The situation is represented by the inequality
$$\pi r^2 + 80\pi r \geq 8r^2 + 128r + 512$$
This inequality is equal to:
$$\pi r^2 - 8r^2 + 80\pi r - 128r - 512 \geq 0$$
$$(\pi - 8)r^2 + (80\pi - 128)r - 512 \geq 0$$

b) Let $f(r) = (\pi - 8)r^2 + (80\pi - 128)r - 512$.
With the parameter a of funtion f being negative, this funtion is positive between the zeros.
Zeros of f:

$-(80\pi - 128) \pm \sqrt{(80\pi - 128)^2 - 4(\pi - 8)(-521)}$,

that is, approximately, 5.22 and 20.16.

Dimensions of the safety cone to the nearest millimetre:
Radius: 5.2 cm to 20.1 cm.
Apothem: 85.2 cm to 100.1 cm.
Side of the base: 26.4 cm to 56.2 cm.

c) No, it is impossible.
Several possible justifications. Example:
The following inequality must be solved:
$(\pi - 8)r^2 + (70\pi - 128)r - 512 \geq 0$.
With the value of parameter a always being negative, the solution is once again found between the zeros of the funtion associated to the left side. However, in this case, the funtion does not have a zero, such as shown in the following calculation of the discriminant:
$(70\pi - 128)^2 - 4(\pi - 8)(-512) \approx -1502$

Practice 4.2 (*cont'd*) — Page 221

13. a) The height of Jim's centre of gravity when he is in standing position.

b) To the nearest millimetre, Jim reaches a height of 2.279 m.

c) To the nearest hundredth of a second, Jim's centre of gravity is above 2.1 m from 0.28 s to 0.66 s.

d) No, Jim was not less than 0.2 s above 2.24 m.

14. a) For all real numbers except for 4.

b) A and B have opposite signs.

c)

Value of x	$]-\infty, 2[$	2	$]2, 4[$	4	$]4, +\infty[$
Sign of $(x - 2)$	−	0	+	+	+
Sign of $(4 - x)$	+	+	+	0	−

The solution set is $]-\infty, 2[\cup]4, +\infty[$.

14. d) 1)

Value of x	$]-\infty, -\frac{1}{2}[$	$-\frac{1}{2}$	$]-\frac{1}{2}, \frac{1}{2}[$	$\frac{1}{2}$	$]\frac{1}{2}, +\infty[$
Sign of $(2x + 1)$	−	0	+	+	+
Sign of $(2x - 1)$	−	−	−	0	+

The solution set is $[-\frac{1}{2}, \frac{1}{2}[$.

2)

Value of x	$]-\infty, -1[$	−1	$]-1, -\frac{1}{2}[$	$-\frac{1}{2}$	$]-\frac{1}{2}, \frac{1}{2}[$	$\frac{1}{2}$	$]\frac{1}{2}, +\infty[$
Sign of $(x + 1)$	−	0	+	+	+	+	+
Sign of $(2x - 1)$	−	−	−	−	−	0	+
Sign of $(4x + 2)$	−	−	−	0	+	+	+

The solution set is $]-1, -\frac{1}{2}[\cup]\frac{1}{2}, -\infty[$.

3)

Value of x	$]-\infty, -4[$	−4	$]-4, \frac{5}{2}[$	$\frac{5}{2}$	$]\frac{5}{2}, 4[$	4	$]4, +\infty[$
Sign of $(5 - 2x)$	+	+	+	0	−	−	−
Sign of $(x + 4)$	−	0	+	+	+	+	+
Sign of $(x - 4)$	−	−	−	−	−	0	+

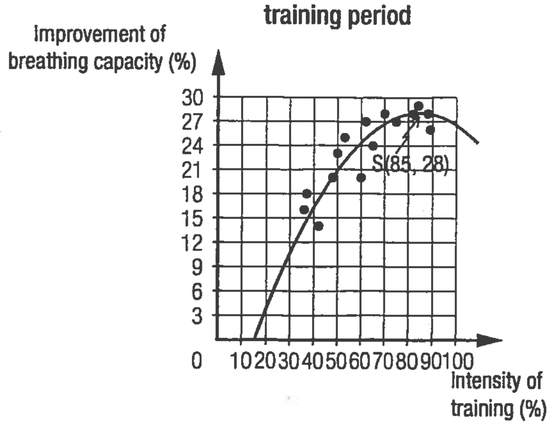
The solution set is $]-\infty, -4] \cup [\frac{5}{2}, 4[$.

SECTION 4.3 Finding the rule

Problem — Page 222

Several possible answers. Example:
The information is represented in a Cartesian plane.
The scatter plot is then modelled by a quadratic function.

Results after a 5-week training period



According to this model, we can estimate from the graph that the increase in breathing capacity is at approximately 27%.

Activity 1

Page 223

a. These functions have the same zeros, 3 and 5, as well as the same axis of symmetry in $x = 4$.

b. *Several possible answers. Example:*
$f(x) = x^2 - 8x + 15$
$g(x) = -6x^2 + 48x - 90$
$h(x) = 3x^2 - 24x + 45$

c. *Several possible conjectures. Example:*
The rule of each function can be written in the form $f(x) = ax^2 - 8ax + 15a$, or $f(x) = a(x^2 - 8x + 15)$ for a certain value of **a**.
Several possible arguments. Example:
All the functions of this family can be written in the form $f(x) = a(x - 4)^2 + k$ since the equation of the axis of symmetry is $x = 4$. Moreover, the graph of each function passes through the point (5, 0).
Therefore $0 = f(5) = a(5 - 4)^2 + k = a + k$.
As a result $k = -a$.
The rule of the function is therefore
$f(x) = a(x - 4)^2 - a = ax^2 - 8ax + 15a$.

d. ① 1 and 3. ② 2 and -1.
③ There is only zero, -3.

e. ① $f_1(x) = 2x^2 - 8x + 6$ ② $f_2(x) = 3x^2 - 3x - 6$
③ $f_3(x) = x^2 + 6x + 9$

f. $-\frac{b}{a}$ is the sum of the zeros, $\frac{c}{a}$ is the product of the zeros.
Let x_1 and x_2, be the zeros of a function. The function could then be written as:
$f(x) = a(x - x_1)(x - x_2)$.
$f(x) = a(x^2 - x_1x - x_2x + x_1x_2)$
$f(x) = ax^2 - a(x_1 + x_2)x + ax_1x_2$.
It can be deduced that $b = -a(x_1 + x_2)$,
therefore $-\frac{b}{a} = (x_1 + x_2)$.
In the same way, , $c = ax_1x_2$, therefore $\frac{c}{a} = x_1x_2$.

Activity 2

Page 224

a. Tourny Fountain
Since the vertex is already known, it is preferable to use the standard form of the function.
$f(x) = a(x + 2)^2 + 3$
x and $f(x)$are then replaced by the coordinates of the point that is already known to determine the value of parameter **a**.
$2.6 = a(-1 + 2)^2 + 3$
$a = -\frac{2}{5}$
By using this value, the following is obtained:
$f(x) = -\frac{2}{5}(x + 2)^2 + 3$.

Barcelona Fountain
Since the zeros are already known, it is preferable to use the factored form of the function.
$f(x) = ax(x - 12)$
x and $f(x)$ are then replaced by the coordinates of the point that is already known to determine the value of parameter **a**.
$8.1 = 3a(3 - 12)$
$a = -0.3$
By using this value, the following is obtained:
$f(x) = -0.3x(x - 12)$.

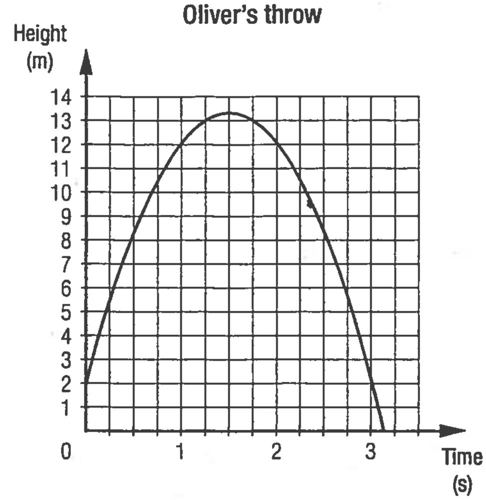
b. $(-2 - \sqrt{7.5}, 0)$, that is, to the nearest centimetre, 4.74 m left of the base of the centre of the fountain.

c. 10.8 m

Activity 3

Page 225

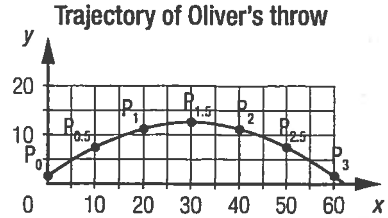
a.



b. The zero is $\frac{-15 - \sqrt{265}}{-10}$, that is, approximately 3.1.
It is the duration of the throw in seconds. The ball touched the ground approximately 3.1 s after it was thrown by Oliver.

c.

Time elapsed (s)	x	y
0	0	2
0.5	10	8.25
1	20	12
1.5	30	13.25
2	40	12
2.5	50	8.25
3	60	2



d. $y = -0.0125(x - 30)^2 + 13.25$ or, in the general form, $y = -0.0125x^2 + 0.75x + 2$.

e. The x-intercept is $30 + 2\sqrt{265}$, that is, approximately 62.56. It represents the horizontal displacement of the ball before touching the ground.

f. The two graphs demonstrate parabolas opening downwards with the same maximum and y-intercept. The graphs differ in the direction of the opening of the parabolas, and therefore in their x-intercepts. Moreover, the first graph demonstrates two variables with different dimensions, one of space and the other of time, yet in the second graph, both are dimensions of space involved.

Technomath

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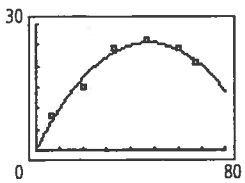
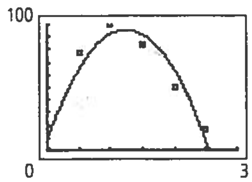
a. This information represents the values of parameters **a**, **b** and **c** of the quadratic curve which best represents the scatter plot in Screen **2**.

b. 1)



2)



c. 1) 9.7278046 2) -4218.95

Practice 4.3

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1.

Function	①	②	③
a)	P = 33 S = -14	P = -15 S = 2	P = 8 S = 6
b)	-11 and -3	-3 and 5	2 and 4

Function	④	⑤	⑥
a)	P = -6 $S = -\frac{5}{2}$	$P = -\frac{1}{2}$ $S = -\frac{1}{12}$	P = 1 S = 4
b)	-4 and $\frac{3}{2}$	$-\frac{3}{4}$ and $\frac{2}{3}$	$2 + \sqrt{3}$ and $2 - \sqrt{3}$

2. a) ① $f_1(x) = \frac{2}{3}x^2 - 4x + 7$ ② $f_2(x) = -\frac{1}{2}x^2 + \frac{7}{2}x - 3$
③ $f_3(x) = \frac{12}{25}x^2 + \frac{36}{25}x - \frac{73}{25}$
b) Curve f_2.

3. a) ① $x = 1$ ② $x = -6$ ③ $x = 2$
b) ① 4 ② 2 ③ $\sqrt{3}$
c) a) $x = \frac{x_1 + x_2}{2}$ b) $d = \frac{x_2 - x_1}{2}$

4. a) Increasing interval: $\left[\frac{7}{2}, +\infty\right[$.
Decreasing interval: $\left]-\infty, \frac{7}{2}\right]$.
b) $f(x) = \frac{1}{2}x^2 - \frac{7}{2}x + 5$
c) $\left[-\frac{9}{8}, +\infty\right[$

Practice 4.3 (*cont'd*)

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5. $f(x) = \frac{3}{8}(x - 4)^2 - 6$

6. a) $f(x) = -\frac{5}{16}x^2 + \frac{5}{2}x$ b) $f(x) = -4x^2 + 80x - 275$
c) $f(x) = -\frac{1}{2}x^2 - 3x - \frac{13}{2}$ d) $f(x) = \frac{3}{4}x^2 - 12$
e) $f(x) = -\frac{3}{16}x^2 - \frac{3}{4}x + 6$ f) $f(x) = x^2 - 4x + 3$
g) $f(x) = -\frac{1}{4}x^2 + 2x - 4$ h) $f(x) = -\frac{5}{4}x^2 + \frac{15}{2}x - \frac{13}{4}$

7. $f_1(x) = -4x^2 + 16x - 12$ $f_2(x) = x^2 + 6x + 5$
$f_3(x) = 2x^2 + 4x - 6$ $f_4(x) = -2x^2 + 6x + 8$
$f_5(x) = \frac{1}{4}(x - 4)^2 - 1$ $f_6(x) = -4(x + 1)^2 + 9$

	f_1	f_2	f_3
Zeros of the function	1 and 3	-5 and -1	-3 and 1
Coordinate of the vertex	(2, 4)	(-3, -4)	(-1, -8)
y-intercept	-12	5	-6

	f_4	f_5	f_6
Zeros of the function	-1 and 4	2 and 6	-2,5 and 0,5
Coordinate of the vertex	$\left(\frac{3}{2}, \frac{25}{2}\right)$	(4, -1)	(-1, 9)
y-intercept	8	3	5

Practice 4.3 (*cont'd*)

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8. a) By placing the intercept at the point of impact of the ball, the following is obtained: $y = -\frac{1}{84}x^2 + \frac{12}{7}x$.
b) To the nearest metre, the ball reached a height of 62 m.

9. a) 8 m
b) Orange trajectory: $y_1 = -0.19x^2 + 5.32x - 18.24$
The vertex of the green trajectory is determined by evaluating the orange trajectory for $x = 8$.
Therefore, S(8,12,16) is found.
Green trajectory: $y_2 = -0.19x^2 + 3.04x$

Practice 4.3 (*cont'd*)

Page 232

10. a) *Several possible answer according to the curve drawn. Example:*
Using the quadratic regression line that can be obtained by using a graphing calculator, the following is determined:
$f(x) = -0.0031x^2 + 0.6359x + 13.401$.
b) The y-intercept corresponds to the height from where the stone is launched from the trebuchet and the x-intercept corresponds to the horizontal distance covered by the projectile (stone).
c) *Several possible answers according to the equation obtained. Example:*
By modelling the situation through the use of the quadratic regression curve, 248 m is found.

11. **a)** $f_1(1) = 2 \cdot 1^2 - 3 \cdot 1 + 1 = 0$;
$f_2(1) = -4 \cdot 1^2 + 1 \cdot 1 + 3 = 0$;
$f_3(1) = 0.5 \cdot 1^2 + 1.5 \cdot 1 - 2 = 0$

b) For f_1: $\frac{1}{2}$ For f_2: $-\frac{3}{4}$ For f_3: -4
Explanation: the value of the second zero is $\frac{c}{a}$ since the product of the zeros is $\frac{c}{a}$ and one of the zeros has a value of 1.

c) 2 and $-\frac{4}{3}$.

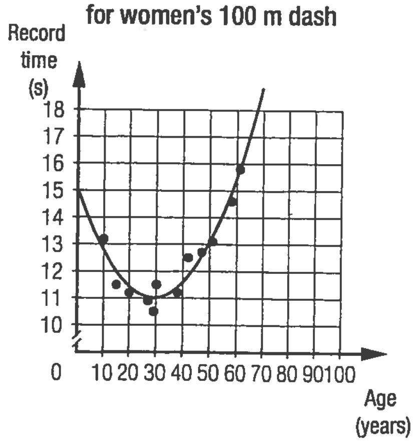
12. **a)** The second zero is −4.
Several possible answers. Example:
The sum of both zeros is -1 because $\frac{-b}{a} = \frac{-k}{k} = 1$.
With one of the zeros being 3, the other zero is therefore -1 − 3.

b) The second zero is $\frac{1}{2}$.
Several possible answers. Example:
The product of the zeros is equal to 1 because $\frac{c}{a} = \frac{k}{k} = 1$.
With the value of the first zero being 2, the second must therefore have a value of 1 ÷ 2.

c) $-\frac{1}{2}$
Several possible answers. Example:
The product of the zeros is equal to k. Since the first zero is 1, the second zero is k. The sum of the zeros is equal to −k. Therefore k + 1 = -k, which allows one to conclude that $k = -\frac{1}{2}$.

Practice 4.3 *(cont'd)* Page 233

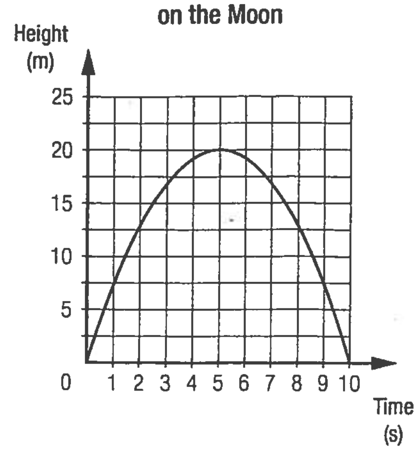
13. **a)** American records set for women's 100 m dash



b) *Several possible answers according to the curve drawn in* ***a).***
By modelling the information through the use of the quadratic regression curve, the following is obtained:
$f(x) = 0.0046x^2 - 0.267x + 14.899$.

c) *Several possible answers according to the equation of the quadratic function found in* ***b).***
According to the equation presented above, to the nearest tenth of a second, 18.7 s is obtained.

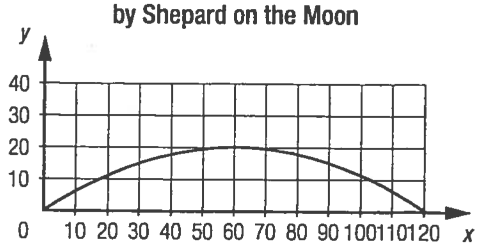
14. **a)** Ball hit by Alan Shepard on the Moon



b) 0 s and 10 s. Respectively, they represent the moment the ball was hit and the moment it touched the Moon's ground.

c) Let x be: the horizontal displacement of the ball (in m);
Let y be: the height of the ball (in m).

Trajectory of the ball hit by Shepard on the Moon



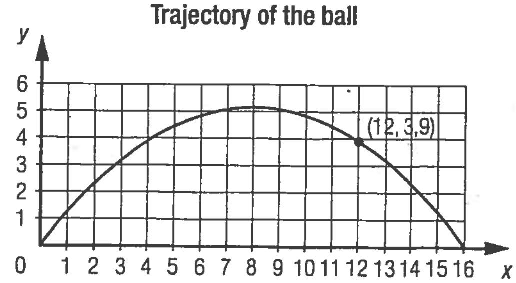
d) $f(x) = -\frac{1}{180}x^2 + \frac{2}{3}x$

e) At 120 m from the astronaut.

Practice 4.3 *(cont'd)* Page 234

15. **a)** Let x be: the horizontal displacement of the ball (in m);
Let y be: the height of the ball (in m) in relation to the point of departure of the throw.

Trajectory of the ball



b) *Several possible answers according to the positioning of the axes. Example:*
For the trajectory drawn above, the equation is:
$y = -\frac{13}{160}x(x - 16)$.

c) 6.8 m

16. **a)** $f(x) = \frac{4}{375}x^2 + 6$

b) At 8.6 cm from the edge of the container.

SECTION 4.4 Distance and the quadratic function

Problem

Page 235

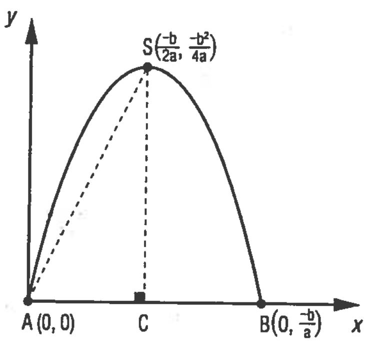
Several possible answers. Example:

There are no restrictions that affect parameter **a**: it can assume all the negative values.

In terms of parameter **b**, the value must be greater than $\sqrt{12}$.

Demonstration

The parabola of the equation $y = ax^2 + bx$ is presented in the Cartesian plane below. Right triangle ACS is then drawn.



Through the use of Pythagorean theorem, the following is obtained:

$(m\,\overline{AS})^2 = (m\,\overline{AC})^2 + (m\,\overline{CS})^2$.

$(m\,\overline{AS})^2 = \left(\frac{-b}{2a}\right)^2 + \left(\frac{-b^2}{4a}\right)^2 = \frac{b^2}{4a^2} + \frac{b^4}{16a^2}$.

$m\,\overline{AB} = \frac{-b}{a}$ is also obtained.

It is supposed that $m\,\overline{AS} > m\,\overline{AB}$. Since $m\,\overline{AS}$ and $m\,\overline{AB}$ are positive numbers, it can be stated that $(m\,\overline{AS})^2 > (m\,\overline{AB})^2$.

Therefore, the following equation is found: $\frac{b^2}{4a^2} + \frac{b^4}{16a^2} > \frac{b^2}{a^2}$.

By dividing each term by $\frac{b^2}{a^2}$, which is a positive number, $\frac{1}{4} + \frac{b^2}{16} > 1$ is obtained.

The solution gives $b^2 > 12$.

Since b is inevitably positive, this is equal to $b > \sqrt{12}$.

Activity 1

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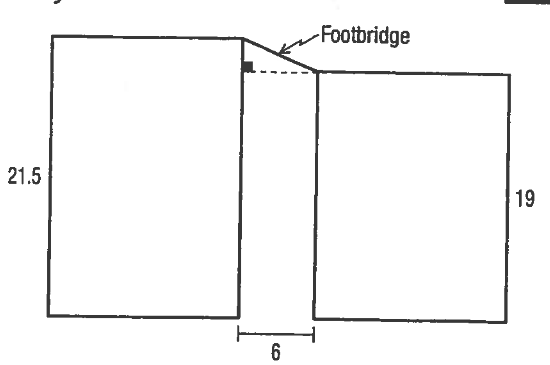
a.



b. It would be possible to install the footbridge perpendicularly to join both buildings.

In this case, the footbridge would measure 6.5 m, that is $\sqrt{(21.5 - 19)^2 + 6^2}$. This therefore represents the minimal length for a foot bridge to join both roofs.

c. The length of this cable is $\sqrt{117}$ m, that is approximately 10.8 m.

d. $m\,\overline{CD} = \sqrt{1^2 + 6^2} = \sqrt{37}$

$m\,\overline{DE} = 7$

$m\,\overline{CE} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

The perimeter is therefore $(17 + \sqrt{37})$ units.

e. Starting from any two points, a right triangle can be drawn in which one of the legs of the triangle is horizontal and measures $x_2 - x_1$ (or the opposite of this expression if $x_1 > x_2$) and the other leg is vertical and measures $y_2 - y_1$ (or the opposite).

Therefore Pythagorean theorem can be applied to obtain:

$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Activity 2

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a. 1) Since $x > 255$, the distance between points A and P is $x - (-255)$. We therefore have $d(A, P) = x + 255$.

2) $d(S, P) = \sqrt{x^2 + 8100}$

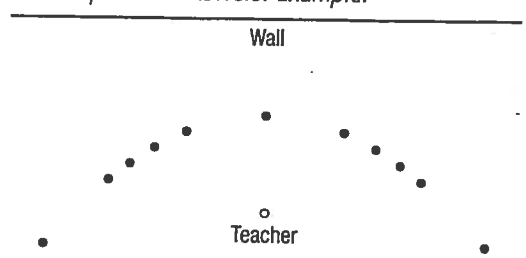
b. $8x^2 - 510x + 7875 = 0$

c. Teresa can head towards any point of the segment whose extremities have $(26\frac{1}{4}, 0)$ and $(37\frac{1}{2}, 0)$, as coordinates, meaning towards a point situated between 26.25 m and 37.5 m on the right of the point closest to her on the road.

Activity 3

Page 238

a. *Several possible answers. Example:*



b. *Several possible answers. Example:*

The curve is symmetrical in relation to the line that is perpendicular to the wall and passes through the position of the teacher. The point that is closest to the wall is situated halfway between the teacher and the wall. The curve has the shape of a parabola.

c. The y-coordinates are respectively 2, 1, and 2.

d. **1)** 5 **2)** $7\frac{1}{4}$ **3)** $21\frac{1}{4}$ **4)** 37

e. The two students are located $4\sqrt{2}$ m from each other, that is approximately 5.66 m.

Practice 4.4 — Page 241

1. d(A, B) = $2\sqrt{5} \approx 4.47$ d(C, D) = $\sqrt{149} \approx 12.21$
   d(E, F) = $\sqrt{45} \approx 6.71$ d(G, H) = 5

2. **a)** Length of the rectangle: $\sqrt{160} = 4\sqrt{10}$
   Width of the rectangle: $\sqrt{40} = 2\sqrt{10}$
   The area is 80 units squared since
   $4\sqrt{10} \times 2\sqrt{10} = 80$.
   b) $12\sqrt{10} \approx 37.95$
   c) d(A, C) = d(B, D) = $10\sqrt{2}$

3. **a)** A(0, 0) B(1, 5) C(2, 8) D(3, 9)
   E(4, 8) F(5, 5) G(6, 0)
   b) 19.35 units
   c) By using more points on the curve.
   Several possible answers. Example:

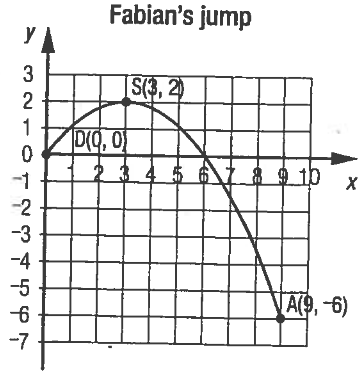
A(0, 0)	H(0.5, 2.75)	B(1, 5)	I(1.5, 6.75)
C(2, 8)	J(2.5, 8.75)	D(3, 9)	K(3.5, 8.75)
E(4, 8)	L(4.5, 6.75)	F(5, 5)	M(5.5, 2.75)
G(6, 0)			

   The length of the trajectory estimated to the nearest hundredth would therefore be 19.45 units.

Practice (cont'd) — Page 242

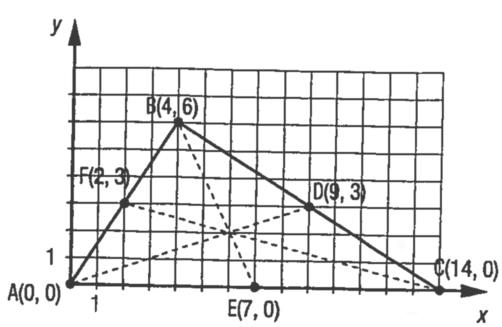
4. **a)** *x*: Fabian's horizontal displacement in the air
   y: Fabian's height in the air in comparison to his starting point.

Trajectory of Fabian's jump



   b) The distance covered is $\sqrt{117}$ m, that is, approximately 10.8 m.
   c) 10 m

5.



   a) m $\overline{AB} = \sqrt{52} = 2\sqrt{13} \approx 7.2$
   m $\overline{BC} = \sqrt{136} = 2\sqrt{34} \approx 11.7$; m $\overline{AC} = 14$
   b) $\overline{AC}$ is the longest side and $\overline{AB}$ is the shortest side.
   c) m $\overline{AD} = \sqrt{90} = 3\sqrt{10} \approx 9.5$
   m $\overline{BE} = \sqrt{45} = 3\sqrt{5} \approx 6.7$
   m $\overline{CF} = \sqrt{153} = 3\sqrt{17} \approx 12$
   d) $\overline{CF}$ is the longest median and $\overline{BE}$ is the shortest median.
   e) Yes, the shortest median is associated to the longest side and the longest median is associated to the shortest side.

6. **a)** Yes.
   d(A, M) = $2\sqrt{5}$ d(B, M) = $2\sqrt{5}$ d(A, B) = $4\sqrt{5}$
   It is noted that d(A, M) = d(B, M) = $\frac{1}{2}$d(A, B).
   b) The midpoint M of $\overline{CD}$ is (3, -7).
   If M is the midpoint, then d(C, M) = d(D, M) = $\frac{1}{2}$d(C, D), which is the case here. In fact:
   m $\overline{CM} = \sqrt{((-9) - 3)^2 + ((-12) - (-7))^2} = 13$
   m $\overline{DM} = \sqrt{(15 - 3)^2 + ((-2) - (-7))^2} = 13$
   m $\overline{CD} = \sqrt{(15 - (-9))^2 + ((-2) - (-12))^2} = 26$

Practice 4.4 (cont'd) — Page 243

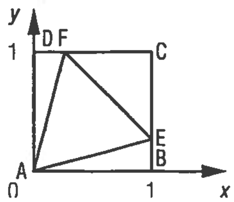
7. **a) 1)** S(250, 50) **2)** (0, −50) and (500, −50).
   b) $y = -\frac{1}{625}(x - 250)^2 + 50$
   c) $(250 - 125\sqrt{2}, 0)$ and $(250 + 125\sqrt{2}, 0)$.
   The distance between these two points is $250\sqrt{2}$ m, that is, approximately 353.6 m.
   d) Approximately 88.7 m.
   e) Approximately 250.2 m.

8. If the position of the pirate ship were to be used as the origin of the Cartesian plane and the units were in metres, the new ride would be situated at point (100, 25).
   Several possible approaches. Example:
   In order for the new ride to be situated at an equal distance from the pirate ship and the merry-go-round, it must be found on the line of the equation $x = 100$, therefore on a point having the coordinates (100, y).
   In order for the new ride to be situated at an equal distance from the pirate ship and the bumper cars, it must be
   $\sqrt{(100 - 60)^2 + (y - 120)^2} = \sqrt{(100 - 0)^2 + (y - 0)^2}$.
   By squaring both sides of the equation, the solution would be $y = 25$.

Practice 4.4 (cont'd) — Page 244

9. The y-coordinate of point B would be 1 or 9.
   Several possible approaches. Example:
   Let (6, y) be the coordinates of point B.
   The equation
   $\sqrt{(6 - 3)^2 + (y - 5)^2} = 5$ must therefore be solved, which comes down to $y^2 - 10y + 9 = 0$. The solutions are both 1 and 9.

10. a) The side of the triangle measures $\sqrt{8 - 4\sqrt{3}}$ units, that is, approximately 1.035 units.
Several possible approaches. Example:
With triangles ADF and ABE being right triangles, and since $m\,\overline{AE} = m\,\overline{AF}$ and that $m\,\overline{AB} = m\,\overline{AD} = 1$, it can be deduced that $m\,\overline{BE} = m\,\overline{DF}$.

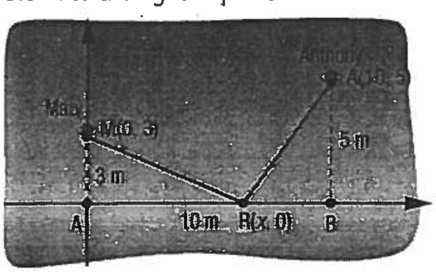


Let r be the measures of these last two segments.
Given, E(1, r) and F(r, 1).
Moreover, d(A, E) = d(F, E).
As a result, there is
$\sqrt{(1 - 0)^2 + (r - 0)^2} = \sqrt{(1 - r)^2 + (r - 1)^2}$,
that comes down to the equation $r^2 - 4r + 1 = 0$.
Only the solution that is less than 1 should be retained, that is $r = 2 - \sqrt{3} \approx 0.268$.
The measure of the side of the triangle is:
$d(A, E) = \sqrt{1 + r^2} = \sqrt{1 + (2 - \sqrt{3})^2} = \sqrt{8 - 4\sqrt{3}} \approx 1.035$.

b) The area of the triangle is $(2\sqrt{3} - 3)$ units squared, that is, approximately 0.464 units squared.
Several possible approaches. Example:
By subtracting, the following is obtained:
Area of Δ AFE = area of square − area of Δ ABE − area of Δ ADF − area of Δ FEC

$$= 1 - \frac{2 - \sqrt{3}}{2} - \frac{2 - \sqrt{3}}{2} - \frac{(1 - (2 - \sqrt{3}))^2}{2}$$
$$= 2\sqrt{3} - 3$$
$$\approx 0.464$$

11. a) 5.8 m to the right of point A.



Several possible approaches. Example:
If the origin in the Cartesian plane were situated at point A, the coordinates of the meeting point R would be (x, 0). One must therefore solve the equation $\sqrt{9 + x^2} = \sqrt{(10 - x)^2 + 25}$, which is simplified to a first-degree polynomial equation whose solution is $x = 5.8$.

b) To the nearest centimetre, the distance that separates them from point A is 6.53 m.

c) To the nearest decimetre, 3.1 m to the right of point A or 9.7 m to the left of point A.
Several possible approaches. Example:
The situation is translated by the equation
$2 \times \sqrt{9 + x^2} = \sqrt{(10 - x)^2 + 25}$, that can be simplified to $3x^2 + 20x - 89 = 0$.
The solution is $x = \frac{-10 \pm \sqrt{367}}{3}$.

11. d) To the nearest centimetre, the total distance required to be covered would be 12.84 m.

e) Yes. For example, at 3.75 m from point A, the total distance needed to be covered, to the nearest centimetre is 12.81 m.

12. a) *Several possible answers. Example:*

b) *Several possible answers according to the representation obtained in* ***a).***
Example:
After t seconds, Patrick will find himself at the point of coordinates (t, 0) and Nancy, at the point of coordinates (30 − t, 0,5).

c) They will be able to touch hands for the first time after 14.4 s.
Several possible approaches. Example:
Patrick and Nancy will be able to touch hands when the distance between them will be 1.3 m.
This corresponds to the equation
$\sqrt{((30 - t) - t)^2 + 0{,}5^2} = 1.3$. This equation can be simplified to $(30 - 2t)^2 = 1.44$. The solution is $t = 15 \pm 0.6$.
The smallest value must be used as a solution.

Practice 4.4 *(cont'd)*

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13. To the nearest centimetre, the height of point A is 75.83 m.

14. a) $c = \frac{1}{4}$

b) d(A, B) = 1

c) 4:1

d) Yes. Points A and B having the same y-coordinate as the focus, they find themselves at the same distance from the directrix as the focus, that is at 2c. According to the definition of the parabola, they are equally situated at a distance of 2c from the focus. It is deduced that the distance between points A and B is always 4c, that is, a distance 4 times longer than the distance between the focus and the vertex of the parabola.

Chronicle of the past

Page 246

1. a) $x^3 = 39x + 200$
Cardano's formula determines that 8 is a solution for this equation.

b) According to the theorem, it is known that the polynomial $x^3 = 39x + 200$ is divisible by $(x - 8)$. By carrying out this division, the following quotient is obtained: $x^2 + 8x + 25$. We therefore have the equation $(x - 8)(x^2 + 8x + 25) = 0$, which implies that $x - 8 = 0$ or $x^2 + 8x + 25 = 0$. Yet the equation $x^2 + 8x + 25 =$ does not have a real solution because its discriminant has a negative value of -36.

Chronicle of the past (*cont'd*)

Page 247

2. a) $(7 + \sqrt{-15})(7 - \sqrt{-15})$ This expression corresponds to a difference of squares.

$49 - (\sqrt{-15})^2$ Once a square root is raised to the power of two, the obtained value is the radicand. In this case, it is therefore -15.

$49 - (-15) = 64$

b) $(2 + 3i)(2 - 3i)$
$4 - 9i^2$
$4 + 9 = 13$

3. a) $y^2 - 13y + 36 = 0$

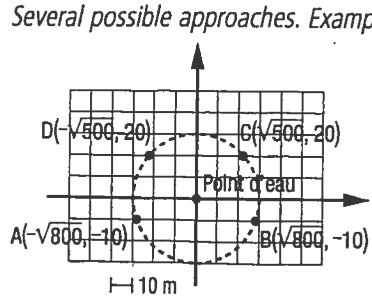
b) $(y - 9)(y - 4) = 0$, where $y = x^2 = 9$ and $y = x^2 = 4$.
The solutions of the equation are -3, -2, 2 and 3.

c) $[-3, -2] \cup [2, 3]$

In the workplace

Page 248

1. *Several possible approaches. Example:*



To facilitate the explanation, a Cartesian plane is used on which the position of the water hole represents the origin. According to the first constraint, the 4 dens must be located 30 m from the water hole; this means that they must be placed on a circle consisting of a 30-m radius and a centre at the origin of the Cartesian plane. It is also known that the dens cannot be positioned less than 20 m from the boundaries of the lot; this means that none of the points representing the dens should have a y-coordinate lower than -10. The location of the first 2 dens are chosen, A and B, by taking into account this limited value of -10 as the *y*-coordinate and the *x*-coordinate of these 2 points is then determined by applying the Pythagorean theorem. The distance between the two dens is therefore $2\sqrt{800}$, that is approximately 56.57 m. To be sure that the location of the last 2 dens is at least 30 m from the first 2, the points found on the circle must have a *y*-coordinate value 30 greater than points A and B (that is *y*-coordinate 20), are chosen. The *x*-coordinate of these points is then calculated by using the Pythagorean theorem, as it had been done for points A and B. The coordinates of points C and D are therefore respectively $(\sqrt{500}, 20)$ and $(-\sqrt{500}, 20)$. Finally, one must verify whether the distance between dens C and D is greater than 30 m, which is the case since it is approximately 44.72 m.

In the workplace (*cont'd*)

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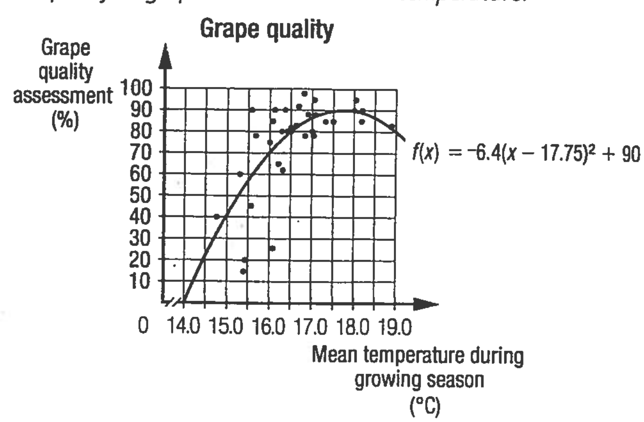
2. *Several possible answers. Models can slightly vary from one student to another. The text must explain, based on the properties of the model such as the vertex, the increasing and decreasing intervals and zeros, the evolution of the quality of grapes in relation to the temperature.*



Overview

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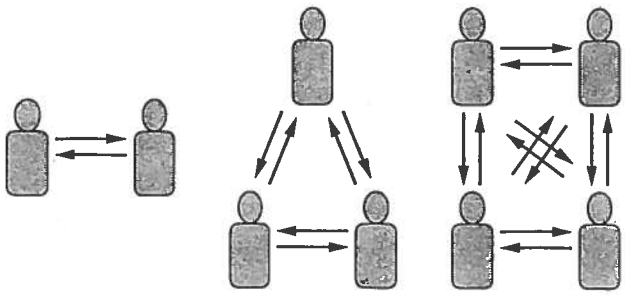
1. a) $y = -2x^2 + 7x - 3$ **b)** $y = 2x^2 + 7x + 3$

c) $y = -2x^2 - 7x - 3$ **d)** $y = 2x^2 + 5x$

e) $y = 2x^2 - 7x + 1$ **f)** $y = 2x^2 - 23x + 66$

2. a) $f(n) = n(n - 1)$
In order for the diagram to correspond to this equation, it must show that the players are playing two games against each person who is participating in the tournament.



b) 20 players.

c) 42, 56, 72 or 90 games have been played.

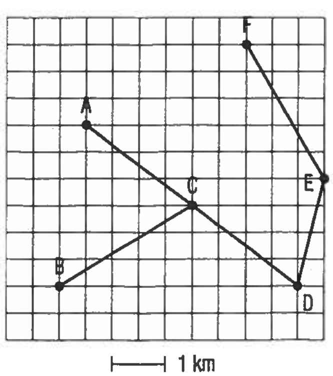
3.

Function	❶	❷
a)	$]-\infty, -2[\cup]2, +\infty[$	$]-\infty, -2[\cup]6, +\infty[$
b)	$\{-2, 2\}$	$\{-2, 6\}$
c)	$]-2, 2[$	$]-2, 6[$

Function	❸
a)	$\left]\frac{-2-\sqrt{5}}{2}, 0\right[\cup \left]0, \frac{-2+\sqrt{5}}{2}\right[$
b)	$\left\{\frac{-2-\sqrt{5}}{2}, 0, \frac{-2+\sqrt{5}}{2}\right\}$
c)	$\left]-\infty, \frac{-2-\sqrt{5}}{2}\right[\cup \left]\frac{-2+\sqrt{5}}{2}, +\infty\right[$

Overview *(cont'd)* Page 251

4. *Several possible answers. Example:*
The path proposed on the plan found below measures 12.89 km.



5. a) 1) After 2 s.
2) The rocket was at 18.75 m, 35 m and 60 m in height.

b) 1) After 6 s.
2) The rocket reached a height of 80 m.

6. a) 1) (2, -1) **2)** (1, 2) **3)** (0, 3)
4) (-1, 2) **5)** (-2, -1)

b) $f(x) = -x^2 + 3$

c) Yes. Parameters **a** from both equations are opposite from each other and parameters **c** are identical.

d) Given the equation $f(x) = ax^2 + bx + c$. The equation of the parabola formed by the position of different vertices when the value of parameter b varies is $y = -ax^2 + c$.
Demonstration
If b = 0, the equation is $y = ax^2 + c$ and the coordinates of the vertex are (0, c).
If b = 1, the equation is $y = ax^2 + x + c$ and the coordinates of the vertex are $\left(-\frac{1}{2a}, -\frac{1}{4a} + c\right)$.
Through the use of symmetry, one can see that the vertex of the parabola formed by the vertices of the equations is (0, c).
Therefore, $y = a_1x^2 + c$. By using point $\left(\frac{1}{2a}, -\frac{1}{4a} + c\right)$, it is determined that $a_1 = -a$ and as a result, the equation of the parabola formed by the set of vertices is $y = -ax^2 + c$.

Overview *(cont'd)* Page 252

7. a) Yes. By placing the origin of the system of axes on the edge of the highest building's roof, the equation of the trajectory is found, that is $y = -\frac{1}{4}(x - 2)^2 + 1$.
The traceur would land at the height of the roof of the building below when $y = -3$. The horizontal displacement of the traceur would therefore be 6 m, which is greater than the distance between the two buildings.

b) $3\sqrt{5}$ m, that is, approximately 6.71 m.

8. a) At 6.36 m.

b) $y = -0.0754(x - 6.36)^2 + 6.1$

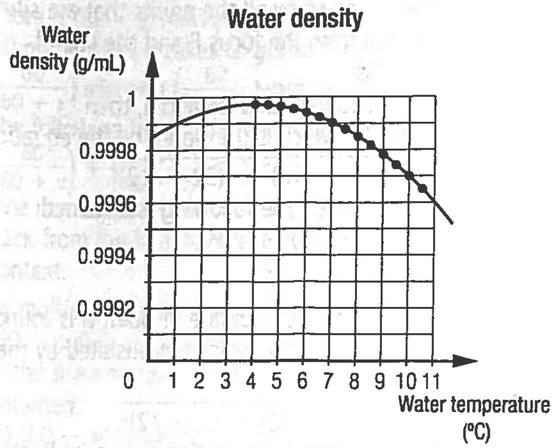
c) To the nearest centimetre, the distance is 9 cm.

Overview *(cont'd)* Page 253

9. a) $(200 + 5n)(1000 - 10n) > 225\,000$
$200\,000 - 2000n + 5000n - 50n^2 > 225\,000$
$-50n^2 + 3000n - 25\,000 > 0$

b) Yes. The number of additional groups of 5 passengers should be between 10 and 50. The plane must carry between 250 and 450 passengers in order for the company to receive over $225,000 in revenue.

10. a)



b) *Several possible answers. Example:*
The quadratic regression curve which is determined by using a graphing calculator gives
$f(x) = -7.290709 \cdot 10^{-6}x^2 + 5.6256743 \cdot 10^{-5} + 0.9998657343$.

c) The water density at 0°C.

d) No. It can be valid for temperatures between 0 to 100°C since outside of this interval, water is no longer in liquid form and it would be inappropriate to assume that the model is always valid. In addition, with the density needing to be greater than 0, one must verify that this is the case throughout the domain of the function. In the case that the water temperature is at 100°C, the density is found to be 0.9326 g/ml, which is a possible value for the density.

Overview (*cont'd*) — Page 254

11. a) Short drop shot: $f(x) = -\frac{1}{4}x^2 + 1$.

Long drop shot: $g(x) = -\frac{1}{27}x^2 - \frac{2}{9}x + 1$.

b) At 2 m.

c) The ball reached $\frac{4}{3}$ m in height, that is, to the nearest cm, 1.33 m.

d) Approximately 2.93 times.

12. a) $x \in]0, 1[$ **b)** $x \in [4, +\infty[$ **c)** $x \in [0, +\infty[$

13. a) $(x - 1)^3$

b) $x \in]11, 1 + 10\sqrt[3]{2}[$, or to the nearest millimetre, between 11.0 and 13.6 mm.

Overview (*cont'd*) — Page 255

14. a) Yes. The bottle will be found at the height of the hot air balloon 0.55 s after Dax's throw.

b) No. The bottle would be found at a minimum 1 m below the level of the hot air balloon, this being 1 s after Dax's throw.

15. a) The throw must be, to the nearest metre and in relation to the wall, at a distance of 0 to 16 m or 34 m to 52 m.

b) The throw must be, to the nearest centimetre and in relation to the wall, at a distance of 26 m to 34 m.

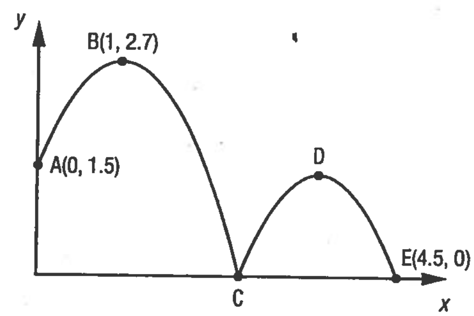
16. a) $f_1(x) = 0.5(x - 4)^2$ $f_2(x) = -0.5(x - 4)^2 + 8$

b) $A(4, 8)$ $B(4 - 2\sqrt{2}, 4)$ $C(4, 0)$ $D(4 + 2\sqrt{2}, 4)$

c) $P = 8\sqrt{6}\ u \approx 19.6\ u$ $A = 16\sqrt{2}\ u^2 \approx 22.63\ u^2$

Bank of problems — Page 256

17. The height reached by the ball on the rebound is 1.2 m.
Several possible answers. Example:
This situation can be represented in a Cartesian plane such as the one below. The required height therefore corresponds to the y-coordinate of point D.

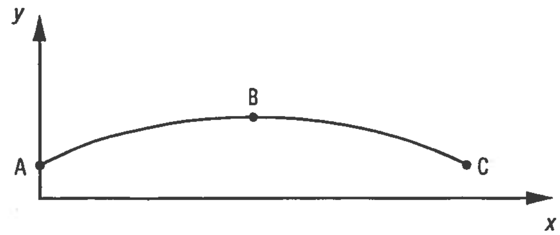


1. The equation of the 1st parabola
Point B is the vertex of this parabola. Therefore
$y = a(x - 1)^2 + 2.7$.
By using the coordinates of point A, the following is obtained: $1.5 = a(0 - 1)^2 + 2.7$.
Therefore, $a = -1.2$.
The equation is $y = -1.2(x - 1)^2 + 2.7$.

2. The x-coordinate of point C
One must replace y by 0 in the previous equation and solve this equation.
$$0 = -1.2(x - 1)^2 + 2.7$$
$$(x - 1)^2 = 2.25$$
$$x = 1 \pm 1.5$$
The positive solution must be used. The x-coordinate of point C is therefore 2.5.

3. The y-coordinate of point D
Since both parabolas have the same opening, both equations have the same parameter **a**. The equation of the 2nd parabola is therefore
$y = -1.2(x - 2.5)(x - 4.5)$.
With point D being the vertex of this parabola, its x-coordinate is 3.5.
The y-coordinate of point D is
$-1.2(3.5 - 2.5)(3.5 - 4.5) = 1.2$.

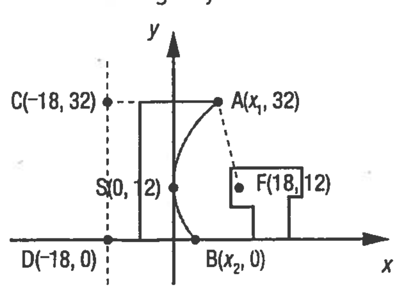
18. *Several possible answers according to the choice of the position of the axes. Example:*
In a Cartesian plane, the trajectory of the ball can be represented in a diagram, such as how it was seen by the farmer.



The x-axis is situated at the height of the edge of the window of the train. The y-axis passes through the area, within the field of vision of the farmer, where the ball is thrown by the person.
A is the position of the ball at the start of the throw, at $t = 0$.
B is the position of the ball in the middle of the throw, at $t = 0.4$.
C is the position of the ball at the end of the throw, at $t = 0.8$.
According to this position of the axes, the y-coordinate of each point of the trajectory corresponds to the height of the ball at different moments of the throw.
y-coordinate of A: $h(0) = -5(0)^2 + 4(0) + 0.2 = 0.2$
y-coordinate of B: $h(0.4) = -5(0.4)^2 + 4(0.4) + 0.2 = 1$
y-coordinate of C: $h(0.8) = -5(0.8)^2 + 4(0.8) + 0.2 = 0.2$
The x-coordinate of point C corresponds to the displacement of the train towards the right during the 0.8 s that the throw lasted. This displacement is 16 m, that is 20×0.8.
The x-coordinate of point B is half of the x-coordinate of C. As a result, the coordinates of the three points are A(0, 0.2), B(8, 1) and C(16, 0.2).
The curve is a parabola in which point B is the vertex.
Its equation is in the form of $y = a(x - 8)2 + 1$.
By using the coordinates of point A, the following is obtained: $0.2 = a(0 - 8)^2 + 1$.
The solution gives $a = \frac{1}{80}$.
The equation of the trajectory is therefore $y = -\frac{1}{80}(x - 8)^2 + 1$.

Bank of problems (*cont'd*) Page 257

19. The distance is approximately 32.2 m.
Several possible approaches. Example:
The situation can be represented in a Cartesian plane in the following way:



The parabola is formed by all the points that are situated at equal distance from the focus F and the line of equation $x = -18$.
Since point A is situated on a parabola, then $d(A, C) = d(A, F)$, which is translated by the equation:
$x_1 - (-18) = \sqrt{(x_1 - 18)^2 + (32 - 12)^2}$.
By squaring both sides, the following is obtained:
$(x_1 + 18)^2 = (x_1 - 18)^2 + 20^2$.
The solution gives $x_1 = \frac{50}{9}$.
In the same way, the x-coordinate of point B is found.
There is $d(B, D) = d(B, F)$, which is translated by the equation:
$x_2 - (-18) = \sqrt{(x_2 - 18)^2 + (0 - 12)^2}$.
By squaring both sides and isolating the variable, the following is obtained: $x_2 = 2$.
The distance between points A and B is
$\sqrt{\left(\frac{50}{9} - 2\right)^2 + (32 - 0)^2} \approx 32.2$.

20. *Several possible conjectures and demonstrations. Example:*
Conjecture
The sum of a positive number and its reciprocal is always greater than or equal to 2.
Demonstration
Let x be a positive real number other than 0.
Its reciprocal is $\frac{1}{x}$.
The solution set of the inequality is determined:
$x + \frac{1}{x} \geq 2$.
Since $x > 0$, each member of this inequality can be multiplied by x without changing the meaning of the inequality. The following equivalent inequalities are therefore obtained:
$$x^2 + 1 \geq 2x$$
$$x^2 - 2x + 1 \geq 0$$
$$(x - 1)^2 \geq 0$$
This last inequality accepts all the values of x as a solution, since the square of a real number is always positive. The solution set of the inequality $x + \frac{1}{x} \geq 2$ is therefore $]0, +\infty[$, which demonstrates the conjecture.

Bank of problems (*cont'd*) Page 258

21. The speed of the current must be less than $20\sqrt{3}$ m/min, that is approximately 34.6 m/min.
Several possible approaches. Example:
In the summer, when there is no current, it takes 1 min for Lee to cover 60 m. His speed is therefore 60 m/min.
Let x be the speed of the current during springtime (in m/min). When Lee advances with the current, his speed in relation to the shore is $(60 + x)$ m/min because the speed of the current is increasing therefore increasing Lee's speed in the water. On the contrary, when he advances against the current, his speed in relation to the