Math 466

Name Key

Solving 2nd Degree Equations/Finding the Zeros

Date

Solve each equation with the quadratic formula.

1) $3x^2 - 12x = 135$

$\{9, -5\}$

$3x^2 - 12x - 135 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-135)}}{2(3)}$

$x = \dfrac{12 \pm \sqrt{1764}}{6}$

$x_1 = \dfrac{12 + 42}{6}$ $\quad$ $x_2 = \dfrac{12 - 42}{6}$

$x_1 = 9$ $\quad$ $x_2 = -5$

2) $2x^2 - 138 = -11x$ $\quad$ $2x^2 + 11x - 138 = 0$

$\{6, -11.5\}$

3) $10x^2 + 4x = 24$

$\{1.362, -1.762\}$

4) $2a^2 - 2a = 19$

$\{3.622, -2.622\}$

5) Find the zeros of the following functions.

a)

$f(x) = \frac{1}{2}(x + 1)^2 - 2$

-3 and 1

$z_1 = h - \sqrt{\frac{-k}{a}}$ $\quad$ $z_2 = h + \sqrt{\frac{-k}{a}}$

$z_1 = -1 - \sqrt{\frac{-(-2)}{1/2}}$

$z_1 = -1 - \sqrt{4}$ $\quad$ $z_2 = h + \sqrt{4}$

$z_1 = -1 - 2$ $\quad$ $z_2 = -1 + 2$

$= -3$ $\quad$ $z_2 = 1$

b)

$f(x) = -3(x - 6)^2 + 12$

4 and 8

Solve each equation by factoring.

6) $n^2 = -21 + 10n$

$\{3, 7\}$

$n^2 - 10n + 21 = 0$

$(n-3)(n-7) = 0$

$n - 3 = 0 \quad n - 7 = 0$

$n = 3 \quad n = 7$

7) $k^2 - 4k = 32$

$\{8, -4\}$

$k^2 - 4k - 32 = 0$

-32

$-8 \quad 4$

$(k-8)(k+4) = 0$

$k - 8 = 0 \quad k + 4 = 0$

$k = 8 \quad k = -4$

8) $5x^2 + 10 = 15x$

$\{1, 2\}$

$5x^2 - 15x + 10 = 0$

$5(x^2 - 3x + 2) = 0$

$5(x-2)(x-1) = 0$

$x - 2 = 0 \quad x - 1 = 0$

$x = 2 \quad x = 1$

9) $4v^2 - 56v = -192$

$\{8, 6\}$

$4v^2 - 56v + 192 = 0$

$4(v^2 - 14v + 48) = 0$

$4(v-8)(v-6) = 0$

$\cancel{4} \quad v - 8 = 0 \quad v - 6 = 0$

$v = 8 \quad v = 6$

10) $2p^2 = 16p$

$\{8, 0\}$

$2p^2 - 16p = 0$

$2p(p-8) = 0$

$p = 0 \quad p - 8 = 0$

$p = 8$

11) $2r^2 - 4r = -2$

$\{1\}$

$2r^2 - 4r + 2 = 0$

$2(r^2 - 2r + 1) = 0$

$2(r-1)(r-1) = 0$

$r - 1 = 0$

$r = 1$

12) $2x^2 = -20 + 13x$

40

$-8 \quad -5$

$2x^2 - 13x + 20 = 0$

$\frac{5}{2}$ or 4

$2x^2 - 8x - 5x + 20 = 0$

$2x(x-4) - 5(x-4) = 0$

$(2x-5)(x-4) = 0$

$2x - 5 = 0$ or $x - 4 = 0$

13) $3n^2 = 23n + 8$

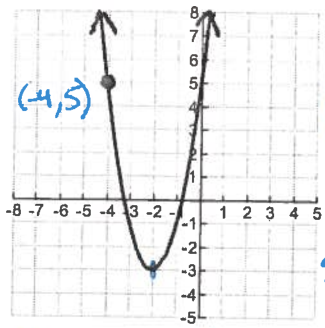
$-\frac{1}{3}, 8$

Math 2 Unit 2.2 Quadratic Word Problems Name: Key

Modeling a Quadratic Function When Given a Graph

Example 1: Write a quadratic function (in vertex form) that models each graph.

a.)



Vertex (-2,-3)

$y=a(x-h)^2+k$

$5=a(-4+2)^2-3$

$5=a(-2)^2-3$

$5=4a-3$

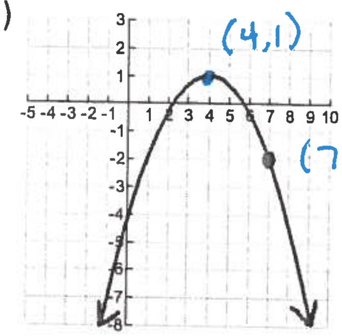
$5+3=4a$

$\frac{8}{4}=\frac{4a}{4}$

$2=a$

$y=2(x+2)^2-3$

b.)



$y=a(x-h)^2+k$

$-2=a(7-4)^2+1$

$-2=a(3)^2+1$

$-2=9a+1$

$-2-1=9a$

$\frac{-3}{9}=\frac{9a}{9}$

$-\frac{1}{3}=a$

$y=-\frac{1}{3}(x-4)^2+1$

Modeling a Quadratic Function Using Various Word Problems

Example 2: Complete each word problem using techniques learned in previous concepts.

a.) Courtney is building a rectangular wading pool. She wants the area of the bottom to be 54 ft^2. She also wants the length of the pool to be 3 ft longer than twice its width. What are the dimensions of the pool?

x, $2x+3$

-108

12, -9

4.5 x 12 ft

$A=2x^2+3x$

$54=2x^2+3x$

$0=2x^2+3x-54$

$0=2x^2+12x-9x-54$

Continued on loose leaf

b.) The formula for throwing a baseball in the air is represented by $h=-16t^2+12t+40$ where h is the height of the ball. After how many seconds will the ball hit the ground?

t = 2 Seconds

Set h=0

Factor, or use Quad. formula

c.) The function $h=-16t^2+1700$ gives an object's height h, in feet, at t seconds.

i.) What does the constant tell you about the height of the object?

height at time = 0

ii.) What does the coefficient of t^2 tell you about the direction the object is moving?

upwards at first

iii.) When will the object be 1000 feet above the ground?

at 6.614 s

iv.) What are a reasonable domain and range for the function h?

Domain [0, 10.31]

Time ball hits the ground

Range [0, 1700]

max height

Example 2 Cont'd: Complete each word problem using techniques learned in previous concepts.

d.) The equation $y = x^2 - 12x + 45$ models the number of books y sold in a bookstore x days after an award-winning author appeared at an autograph-signing reception. What was the first day that at least 100 copies of the book were sold?

$100 = x^2 - 12x + 45$

$0 = x^2 - 12x + 45 - 100$

$0 = x^2 - 12x - 55$ (−55: −11, 5)

$0 = x^2 - 12x + 36 - 36 - 55$

$0 = (x-6)^2 - 91$

$0 = (x - 6 + \sqrt{91})(x - 6 - \sqrt{91})$

$0 = (x + 3.54)(x - 15.54)$

$0 = x + 3.54$ $\quad$ $0 = x - 15.54$

$-3.54 = x$ $\quad$ $15.54 = x$

16th day (78 on day 15)

$y = x^2 - 12x + 45$

$y = (16)^2 - 12(16) + 45$

$y = 109$ books

or $(x-6)^2 - 91 = 0$

$(x-6)^2 = 91$

$x - 6 = \pm\sqrt{91}$

$x = 6 + \sqrt{91}$ or $x = 6 - \sqrt{91}$

$x = 15.54$ or $x = -3.54$

e.) A ball is thrown into the air with an initial upward velocity of 48 ft/s. It height *h* in feet after *t* seconds is given by the function $h(t) = -16t^2 + 48t + 4$.

i.) What height will the ball be when 2 seconds has passed? $t = 2$

$h(t) = -16t^2 + 48t + 4$

$h(2) = -16(2)^2 + 48(2) + 4$

$h(2) = 36$ ft

ii.) In how many seconds will the ball reach its maximum height? Vertex

$h = \frac{-b}{2a}$ $\quad$ $h = 1.5$

$h = \frac{-48}{2(-16)}$

1.5 seconds

iii.) What is the ball's maximum height?

$y = -16t^2 + 48t + 4$

$k = -16(1.5)^2 + 48(1.5) + 4$

$k = 40$ $\quad$ 40 ft

Name: ____________________
Date: ______________ Period: ____

Quadratic Formula Word Problems

1. Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, where t is the time in seconds and h is the height in feet.

   a. How long did it take for Jason to reach his maximum height?

   $h = \frac{-b}{2a}$ $h = \frac{-16}{2(-16)}$ 0.5 Seconds

   $h = 0.5$

   b. What was the highest point that Jason reached?

   $y = -16t^2 + 16t + 480$

   $k = -16(0.5)^2 + 16(0.5) + 480$

   $k = 484$ ft

   c. Jason hit the water after how many seconds?

   $y = -16t^2 + 16t + 480$ $y = 0$

   $y = -16(t^2 - t - 30)$ -30: -6, 5

   $0 = -16(t-6)(t+5)$

   $t - 6 = 0$ or $t + 5 = 0$

   $t = 6$ or $t = -5$

   After 6 seconds

2. If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height *h* after *t* seconds is given by the equation $h(t) = -16t^2 + 128t$ (if air resistance is neglected).

   a. How long will it take for the rocket to return to the ground? $y = 0$

   $0 = -16t^2 + 128t$ $t - 8 = 0$

   $0 = -16t(t-8)$ $t = 8$ After 8 seconds

   b. After how many seconds will the rocket be 112 feet above the ground? $y = 112$

   $112 = -16t^2 + 128t$

   $16t^2 - 128t + 112 = 0$ $t - 7 = 0$ or $t - 1 = 0$

   $16(t^2 - 8t + 7) = 0$ $t = 7$ $t = 1$

   $16(t-7)(t-1) = 0$ at 1 and 7 seconds

   c. How long will it take the rocket to hit its maximum height?

   $h = \frac{-b}{2a}$ $h = 4$

   $h = \frac{-128}{2(-16)}$ at 4 seconds

   d. What is the maximum height? K value

   $y = -16t^2 + 128t$

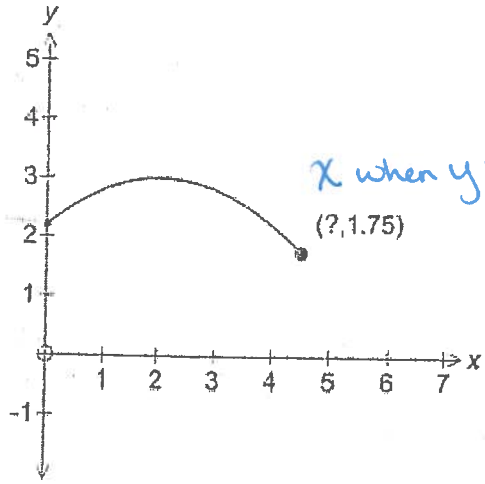
   $k = -16(4)^2 + 128(4)$ 256 ft

   $k = 256$

Question 3

David and Eddie throw a ball to each other during a softball game. The side view of the trajectory of the throw is shown in the graph below. Given the rule f(x) for the trajectory, how far apart are David and Eddie?

$f(x) = -0.2(x-2)^2 + 3$



x when y=1.75

$y = -0.2(x-2)^2 + 3$

$1.75 = -0.2(x-2)^2 + 3$

$0 = -0.2(x-2)^2 + 3 - 1.75$

$0 = -0.2(x-2)^2 + 1.25$

$0.2(x-2)^2 = 1.25$

$(x-2)^2 = \frac{1.25}{0.2}$

$(x-2)^2 = 6.25$

$x-2 = \pm 2.5$

$x = 2.5+2$ $\quad$ $x = -2.5+2$

$\boxed{x = 4.5}$ or $x = -0.5$

or

$z = h \pm \sqrt{\frac{-k}{a}}$

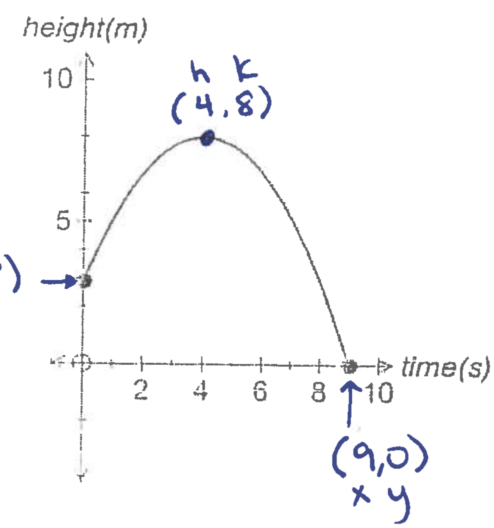
$z = 2 \pm \sqrt{\frac{-1.25}{-0.2}}$

$z_1 = 2 + 2.5$ or $z_2 = 2 - 2.5$

$\boxed{z_1 = 4.5}$ $\quad$ $z_2 = -0.5$

Question 4

From a balcony, Eric launches a projectile along a parabolic path. After four seconds it reaches a maximum height of 8 meters, and hits the ground 5 seconds later. How high is the balcony it was launched from?



$y = a(x-h)^2 + k$

$0 = a(9-4)^2 + 8$

$0 = a(5)^2 + 8$

$\frac{-8}{25} = \frac{a(\cancel{25})}{\cancel{25}}$

$\frac{-8}{25} = a$

$y = \frac{-8}{25}(x-4)^2 + 8$ $\quad$ set $x = 0$

$y = \frac{-8}{25}(0-4)^2 + 8$

$y = \frac{-8}{25}(16) + 8$ $\quad$ 2.88m

$y = 2.88$

$0 = 2x^2 + 12x - 9x - 54$

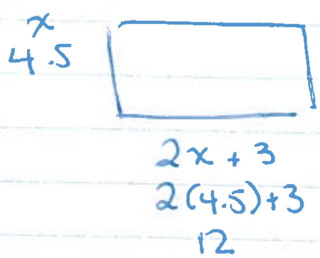
$0 = 2x(x+6) - 9(x+6)$

$0 = (2x-9)(x+6)$

$2x - 9 = 0$ or $x + 6 = 0$

$\frac{2x}{2} = \frac{9}{2}$ $\qquad x = -6$

$\boxed{x = 4.5}$ $\qquad$ impossible width



Pool is 4.5 × 12 ft

b) $h = -16t^2 + 12t + 40$ $\qquad y = 0$

$0 = -16t^2 + 12t + 40$

-40: -8, 5

Zero by Quad. formula or Factor

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-12 \pm \sqrt{12^2 - 4(-16)(40)}}{2(-16)}$

$x = \frac{-12 \pm \sqrt{2704}}{-32}$

$x_1 = \frac{-12 + 52}{-32}$ or $x_2 = \frac{-12 - 52}{-32}$

$x_1 = -1.25$ $\qquad x_2 = 2$

X

$0 = -4(4t^2 - 3t - 10)$

$0 = -4(4t^2 - 8t + 5t - 10)$

$0 = -4(4t(t-2) + 5(t-2))$

$0 = -4[(4t+5)(t-2)]$

$4t + 5 = 0$ or $t - 2 = 0$

$4t = -5$ $\qquad t = 2\ s$ ✓

$t = \frac{-5}{4}$

X

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

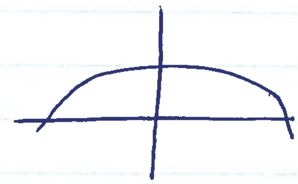
[illegible]

[illegible]

c) $h = -16t^2 + 1700$

i) Constant is the initial value / height at time $= 0$

ii) negative, so ball is moving upwards initially



iii) $h = 1000$

$$h = -16t^2 + 1700$$
$$1000 = -16t^2 + 1700$$
$$0 = -16t^2 + 1700 - 1000$$
$$0 = -16t^2 + 700$$
$$16t^2 = 700$$
$$t^2 = \frac{700}{16}$$
$$\sqrt{t^2} = \sqrt{43.75}$$

$t = \pm 6.614$ seconds

iv) Ball hits the ground when $y = 0$

$$h = -16t^2 + 1700$$
$$0 = -16t^2 + 1700$$
$$16t^2 = 1700$$
$$t^2 = 106.25$$

$t = \pm 10.31$ seconds

Domain : $[0, 10.31]$ Sec.

Max height $h = \frac{-b}{2a}$

$h = \frac{-0}{2(-16)}$

* vertex is $x = 0$, max height is 1700

Range : $[0, 1700]$