Group: _____ Date: _____

Key

Objective 4.2 To apply the properties of exponents

4

1. The first property of exponents is:

   The exponent of the product of two powers of the same base is the sum of the powers' exponents.

   $$m^a \cdot m^b = m^{a+b}$$

   Complete.

   a) $a^2 \cdot a^4 = a^{2+4} =$ a^6

   b) $x^3 \cdot x^2 = x^{3+2} =$ x^5

   c) $x^m \cdot x^n =$ x^{m+n}

   d) $b^3 \cdot b^4 = b^{3+4} =$ b^7

   e) $m^5 \cdot m^2 = m^{5+2} =$ m^7

   f) $10^m \cdot 10^n =$ 10^{m+n}

2. Apply the property of exponents to simplify these expressions.

   a) $7^5 \times 7^2 =$ 7^7

   b) $(-3)^2 \times (-3)^3 =$ $(-3)^5$

   c) $8^3 \times 8^2 =$ 8^5

   d) $6^3 \times 6^{-2} =$ $6^{3-2} = 6^1$

3. Simplify these expressions.

   a) $10^2 \times 10^3 =$ 10^5

   b) $10 \times 10^2 =$ 10^3

   c) $10^{-1} \times 10^3 =$ $10^{-1} \rightarrow \frac{1}{10}$

   d) $10^{-5} \times 10^{-4} =$ $10^{-9} \rightarrow \frac{1}{10^9}$

4. Write each expression as a monomial.

   a) $(2b)^a \cdot (2b)^m =$ $(2b)^{a+m}$ or $2^{a+m} b^{a+m}$

   b) $(-a)^b \cdot (-a)^c =$ $(-a)^{b+c}$

   c) $m^x \cdot m^y =$ m^{x+y}

   d) $(ab)^m \cdot (ab)^n =$ $(ab)^{m+n}$ or $a^{m+n} b^{m+n}$

5. Reminder: The exponent of the quotient of two powers of the same base is the difference in the powers' exponents.

   $$m^a \div m^b = m^{a-b}$$

   Complete.

   a) $a^5 \div a^2 = a^{5-2} =$ a^3

   b) $b^m \div b^n =$ b^{m-n}

   c) $m^x \div m^y =$ m^{x-y}

   d) $x^4 \div x =$ x^3

   e) $x^9 \div x^3 =$ $x^{9-3} = x^6$

   f) $y^6 \div y^2 =$ y^4

6. Apply the property of exponents to simplify the following expressions.

   a) $12^7 \div 12^5 =$ 12^2

   b) $(-3)^4 \div (-3)^3 =$ $(-3)^1$

   c) $8^2 \div 8^{-2} =$ $8^{2-(-2)} = 8^4$

   d) $(-5)^6 \div (-5)^2 =$ $(-5)^4$

7. Simplify these expressions.

a) $10^8 \div 10^3 =$ $10^{8-3} = 10^5$

b) $10^9 \div 10^4 =$ 10^5

c) $10^9 \div 10^3 =$ 10^6

d) $10^{10} \div 10^8 =$ 10^2

8. Write each expression as a monomial.

a) $(ab)^8 \div (ab)^4 =$ $(ab)^4$ or a^4b^4

b) $(2b)^3 \div (2b)^1 =$ $(2b)^2$ or 2^2b^2

c) $(-2b)^5 \div (-2b)^3 =$ $(-2b)^2$ $(-2)^2b^2$

d) $(mn)^2 \div (mn)^{-3} =$ $(mn)^5$ or m^5n^5

9. Simplify.

a) $3x \cdot 8x^3 =$ $24x^4$

b) $16a \cdot 3a^4 =$ $48a^5$

c) $10m^3 \cdot 6m^8 =$ $60m^{11}$

d) $8y^5 \cdot 2y^3 =$ $16y^8$

10. Express the product in scientific notation.

a) $(2.3 \times 10^4) \times (5 \times 10^{-2}) =$ 1.15×10^3

b) $(1.42 \times 10^{-5}) \times (2 \times 10^{-3}) =$ 2.84×10^{-8}

c) $(4 \times 10^3) \times (7.2 \times 10^{-1}) =$ 2.88×10^3

d) $(8 \times 10^5) \times (9 \times 10^2) =$ 7.2×10^8

11. Express the results in exponential form.

a) $5^2 \times 5^3 \times 5^{-4} =$ $5^{2+3-4} = 5^1$

b) $8^9 \div 8^2 \times 8^3 =$ 8^{10}

c) $15^4 \times 15^2 \div 15^3 =$ $15^{4+2-3} = 15^3$

d) $6^{-2} \times 6^5 \div 6^{-4} =$ 6^7

e) $10^{-5} \times 10^2 \div 10^{-3} =$ $10^0 = 1$

f) $(7^2 \times 7^6) \div (7^{-3} \times 7^3) =$ 7^6

g) $(5^2 \div 5) \times (5^8 \div 5^4) =$ 5^5 5×5^4

h) $10^7 \times 10^{-3} \times 10^{-2} =$ 10^2

12. Calculate the value of each expression.

a) $5^4 \times 5^2 =$ $5^6 \rightarrow 15625$

b) $2^3 \times 2^2 =$ 2^5

c) $(2 \times 10^3) + (4 \times 10^2) =$ 2000 400 2400 or 2.4×10^3

d) $(2 \times 10^4) \div (5 \times 10^2) =$ 4×10^1

e) $(3 \times 10^2) \times (6 \times 10^4) =$ 300×60000 1.8×10^7

f) $(7 \times 10^4) - (3 \times 10^3) =$ $7 \times 10^4 - 0.3 \times 10^4$ 6.7×10^4

13. Simplify to a monomial.

a) $4x^3 \cdot 6x^2 \div -12x =$ $-2x^4$

b) $6x^5 \cdot 3x^{-2} \cdot x^4 =$ $18x^7$

c) $10a^3 \div 5a \cdot 4a^5 =$ $8a^7$

d) $3a^{-5} \cdot 8a^3 \div 6a^{-2} =$ $4a^0$ or 4

14. Find the missing term.

a) $12^7 = 12^3 \times$ 12^4

b) $4^5 \times$ 4^{-3} $= 4^2$

c) $a^{16} \div$ a^{12} $= a^4$

d) $m^8 =$ m^6 $\times m^2$

15. True or false?

a) $6^3 \times 6^4 = 6^{12}$ F

b) $3^{10} \times 3^5 = 9^{15}$ F

c) $10^3 \times 10^5 = 10^8$ T

d) $3^8 \times 4^8 = 7^8$ F

e) $a^{16} \div a^8 = a^2$ F

f) $(-3)^4 \times (-3)^{-3} = -3$ T

ACTIVITY 2 Multiplying two powers with the same base

a) Perform the following calculations and express the result in exponential form.

1. $2^3 \times 2^2 =$ 32 $= 2^{5}$

2. $(-2)^2 \times (-2)^3 =$ -32 $= (-2)^{5}$

b) 1. Complete the following demonstration.

$$a^3 \times a^2 = \underbrace{a \times a \times a}_{3 \text{ factors}} \times \underbrace{a \times a}_{2 \text{ factors}}$$

$$= \underbrace{a \times a \times a \times a \times a}_{5 \text{ factors}}$$

$$= a^{5}$$

$$= a^{3+2}$$

2. Explain the procedure for determining the product of two powers with the same base.

To multiply, add the exponents on a common base.

c) Express each result using exponential form.

1. $2^5 \times 2^{10} =$ 2^{15}

2. $(-3)^4 \times (-3)^6 =$ $(-3)^{10}$

3. $a^4 \times a^2 =$ a^{6}

4. $b^m \times b^n =$ b^{m+n}

MULTIPLYING TWO POWERS WITH THE SAME BASE

$$a^m \times a^n = a^{m+n}$$

Add the exponents.

Ex.: $2^2 \times 2^4 = 2^6$; $(-2)^2 \times (-2)^4 = (-2)^6$.

1. Use the law of exponents for multiplying two powers with the same base and express your result in exponential form.

a) $2^4 \times 2^6 =$ 2^{10}

b) $(-3)^2 \times (-3)^5 =$ $(-3)^{7}$

c) $2^5 \times 2 =$ 2^{6}

d) $2^3 \times 2^2 \times 2^4 =$ 2^{9}

e) $\left(-\frac{2}{3}\right)^3 \times \left(-\frac{2}{3}\right)^2 =$ $\left(\frac{-2}{3}\right)^{5}$

f) $\left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^2 =$ $\left(\frac{3}{4}\right)^{4}$

2. Simplify using the law of exponents for the product of two powers with the same base.

a) $x^5 \cdot x^3 =$ x^{8}

b) $a^4 \cdot a =$ a^{5}

c) $m^3 \cdot m^4 \cdot m^2 =$ m^{9}

d) $a \cdot a =$ a^{2}

e) $x^2y \cdot x^4y^2 =$ $x^{6}y^{3}$

f) $a^0b^4 \cdot a^3b^2 =$ $a^{4}b^{6}$

g) $a^m \cdot a^n =$ a^{m+n}

h) $x^p \cdot x^q \cdot x^r =$ x^{p+q+r}

i) $(a^3b^2)(a^5b)(a^2b^4) =$ $a^{10}b^{7}$

j) $a^5 \cdot a \cdot a^6 =$ a^{12}

k) $x^5 \cdot x^5 =$ x^{10}

l) $(mn)(mn) =$ m^2n^2

3. Use the commutative property of multiplication to simplify the following expressions.
(Ex.: $2x^5 \cdot 3x^2 = 2 \cdot 3 \cdot x^5 \cdot x^2 = 6x^7$)

a) $3m^2 \cdot 2m^4 =$ $6m^6$ b) $-5a^4 \cdot 3a^2 =$ $-15a^6$ c) $(4x^3)(-x^5) =$ $-4x^8$

d) $(3x^2y^5)(-2x^3y^2) =$ $-6x^5y^7$ e) $\left(\frac{3}{4}x^2\right)\left(-\frac{8}{9}x^{12}\right) =$ $-\frac{2}{3}x^{14}$ f) $\left(\frac{4}{5}a^5b^2\right)\left(\frac{15}{16}ab^4\right) =$ $\frac{3}{4}a^6b^6$

g) $(3x^2y)(-xy^4) =$ $-3x^3y^5$ h) $\left(-\frac{4}{5}m^4n^2\right)\left(-\frac{5}{2}mn^6\right) =$ $2m^5n^8$ i) $(2x^5)(-3x^2)(5x) =$ $-30x^8$

j) $(5x)(-x) =$ $-5x^2$ k) $(4x)(-2x) =$ $-8x^2$ l) $\left(-\frac{1}{2}mn\right)\left(\frac{4}{5}mn\right) =$ $-\frac{2}{5}m^2n^2$

ACTIVITY 3 Quotient of two powers with the same base

a) Calculate the following and express each result using exponential form.

1. $\frac{2^5}{2^3} =$ 4 $= 2^{2}$ 2. $\frac{2^4}{2^4} =$ 1 $= 2^{0}$ 3. $\frac{3^2}{3^4} =$ $\frac{1}{9}$ $= \frac{1}{3^{2}}$

b) Consider the following quotient $\frac{a^m}{a^n}$.

1. Complete the following demonstrations.

1st case: $m > n$

$$\frac{a^5}{a^2} = \frac{\overbrace{a \times a \times a \times a \times a}^{5 \text{ factors}}}{\underbrace{a \times a}_{2 \text{ factors}}} = \underbrace{a \times a \times a}_{3 \text{ factors}} = a^{5-2}$$

2nd case: $m = n$

$$\frac{a^3}{a^3} = \frac{\overbrace{a \times a \times a}^{\square \text{ factors}}}{\underbrace{a \times a \times a}_{\square \text{ factors}}} = a^{0} = 1$$

3rd case: $m < n$

$$\frac{a^2}{a^5} = \frac{\overbrace{a \times a}^{\square \text{ factors}}}{\underbrace{a \times a \times a \times a \times a}_{\square \text{ factors}}} = \frac{1}{a^{3}} = \frac{1}{a^{5-2}}$$

2. Explain the procedure for determining the quotient of two powers with the same base.

If the base is the same, to divide we subtract the exponents

c) Express each result using exponential form.

1. $\frac{2^8}{2^3} =$ 2^5 2. $\frac{(-3)^5}{(-3)^2} =$ $(-3)^3$ 3. $\frac{3^4}{3^4} =$ 3^0 4. $\frac{3^2}{3^5} =$ $\frac{1}{3^3}$

1.7 Negative exponents

key

ACTIVITY 1 Negative exponents

When the base a is a non-zero real number and n is a natural number, a^{-n} is a negative power defined by $a^{-n} = \frac{1}{a^n}$.

Calculate.

a) $3^{-2} = \frac{1}{3^2}$ b) $(-5)^{-2} = \frac{1}{(-5)^2}$ or $\frac{1}{25}$ c) $-5^{-2} = -\frac{1}{5^2}$ or $-\frac{1}{25}$

d) $(-2)^{-3} = \frac{1}{(-2)^3}$ e) $10^{-4} = \frac{1}{10^4}$ f) $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2}$

g) $(0.2)^{-1} = \frac{1}{0.2} = 5$ h) $(0.05)^{-3} = \left(\frac{1}{20}\right)^{-3} = 20^3$ i) $(0.01)^{-2} = \frac{1}{100}$ or $\frac{1}{10^2}$

$\left(\frac{1}{5}\right)^{-1} = 5$

NEGATIVE EXPONENTS

For any non-zero real number a, and any natural number n, we have:

$$a^{-n} = \frac{1}{a^n}$$

Ex.: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$; $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$; $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{-1}{8}$.

1. Explain why it is necessary for the base a to be non-zero in the expression a^{-n}.

---

2. The laws of exponents (section 1.6) remain valid when the exponents are negative. Simplify the following calculations using the appropriate law of exponents.

a) $2^3 \times 2^{-5}$

2^{3+-5}

$2^{-2} \rightarrow \frac{1}{2^2}$ or $\frac{1}{4}$

b) $(2^2)^{-3}$

$2^{-6} \rightarrow \frac{1}{2^6}$

c) $(x^2y^{-3})^{-2}$

$x^{-4}y^6 \rightarrow \frac{y^6}{x^4}$

d) $\left(\frac{x^{-3}}{y^2}\right)^{-2}$

$\frac{x^6}{y^{-4}}$

$x^6 y^4$

$\left(\frac{y^2}{x^{-3}}\right)^2$

$(x^3y^2)^2$

x^6y^4

e) $\left(\frac{3a^{-2}}{b^4}\right)^{-2}$

$\frac{3a^4}{b^{-8}}$

$3a^4b^8$

Algebra 1 ID: 1

Assignment

Name _____

Date _____ Period ____

Simplify. Your answer should contain only positive exponents.

1) $u^{-2} \cdot 3u^{-4}v^2$

$3u^{-6}v^2 \rightarrow \frac{3v^2}{u^6}$

2) $2y^{-3} \cdot x^2 \cdot 2x^{-2}$

$4y^{-3} \rightarrow \frac{4}{y^3}$

3) $2x^3y^3 \cdot y^4 \cdot 3y^4$

$6x^3y^{11}$

4) $4a^2b^2 \cdot 3ab$

$12a^3b^3$

5) $x^2y^{-2} \cdot 4x^4y^3$

$4x^6y^1$

6) $2a^{-4} \cdot 4ab^{-2}$

$8a^{-3}b^{-2} \rightarrow \frac{8}{a^3b^2}$

7) $x^0 \cdot 4x^4y^2$

$4x^4y^2$

8) $a^{-1}b^0 \cdot 4a^{-3}b^{-4}$

$4a^{-4}b^{-4} \rightarrow \frac{4}{a^4b^4}$

9) $y^{-4}x^4$ $\frac{x^4}{y^4}$

10) $2m^2n^{-3} \cdot 2m^3n^{-1} \cdot 4m^4n^{-1}$

$16m^9n^{-5} \rightarrow \frac{16m^9}{n^5}$

11) $x^4y^{-4} \cdot x^{-2}y^4$

x^2

12) $m^3 \cdot m^{-1}n^0$

m^2

13) $uv^3 \cdot u^4v^0$

u^5v^3

14) $x^4y^{-3} \cdot xy^{-1}$

$x^5y^{-4} \rightarrow \frac{x^5}{y^4}$

15) $2xy^{-4} \cdot yx^3$

$2x^4y^{-3} \rightarrow \frac{2x^4}{y^3}$

16) $4u^3v^{-3} \cdot 2uv^{-4} \cdot 2u^2v^{-4}$

$16u^6v^{-11} \rightarrow \frac{16u^6}{v^{11}}$

17) $2x^4y^4 \cdot 2x^{-1}$

$4x^3y^4$

18) $2x^{-4} \cdot 2x^{-3}y^4$

$4x^{-7}y^4 \rightarrow \frac{4y^4}{x^7}$

19) $2b^{-3} \cdot 2a^{-2}b^{-2}$

$4b^{-7} \rightarrow \frac{4}{b^7}$

20) $a^2b^4 \cdot 2a^0$

$2a^2b^4$

key

monomials
division

#1

Example:

$\frac{(4x^3y^5)}{(2x^2y)} =$

1. Divide the coefficents, or reduce the fraction.

$\frac{(4x^3y^5)}{(2x^2y)} = \frac{4}{2} = 2$

2. Divide the variables by subtracting the exponents.

$\frac{(x^3)}{(x^2)} = x^{3-2} = x \qquad \frac{(y^5)}{(y)} = y^{5-1} = y^4$

Answer: $2xy^4$

Divide the monomials.

1. $\frac{6x^{10}y}{2x^4y} = 3x^6$

2. $\frac{4x^3y^8}{12xy^4} = \frac{1x^2y^4}{3}$

3. $\frac{5x^4y^4}{1x^2y^1} = 5x^2y^3$

4. $\frac{6x^5y}{3x^2} = 2x^3y$

5. $\frac{9x^3y^2}{3x^3y} = 3y^1$

6. $\frac{8x^3y^9}{12y^4} = \frac{2x^3y^5}{3}$

7. $\frac{12x^8y^2}{4xy^2} = 3x^7$

8. $\frac{10xy^7}{12xy^2} = \frac{5y^5}{6}$

9. $\frac{4x^5y^{10}}{4x^4y^4} = xy^6$

10. $\frac{x^5y^3}{x^4y} = xy^2$

Key

monomials
division

#2

Example:

$$\frac{(4x^3y^5)}{(2x^2y)} =$$

1. Divide the coefficents. or reduce the fraction

$$\frac{(4x^3y^5)}{(2x^2y)} = \frac{4}{2} = 2$$

2. Divide the variables by subtracting the exponents.

$$\frac{(x^3)}{(x^2)} = x^{3-2} = x \qquad \frac{(y^5)}{(y)} = y^{5-1} = y^4$$

Answer: $2xy^4$

Divide the monomials.

1. $\frac{4x^4y^3}{2x^2y} = \frac{2x^2y^2}{}$

2. $\frac{6xy}{18y} = \frac{x}{3}$

3. $\frac{9x^7y^8}{18x^6y^5} = \frac{1xy^3}{2}$

4. $\frac{8x^3y^3}{2x^2y^3} = 4x$

5. $\frac{15x^4y}{25xy} = \frac{3x^3}{5}$

6. $\frac{16x^4y^5}{4x^3y^2} = 4xy^3$

7. $\frac{4y^8}{12y^7} = \frac{y}{3}$

8. $\frac{5x^8y^2}{1xy^2} = 5x^7$

9. $\frac{6x^3y^{10}}{3x^2y^5} = 2xy^5$

10. $\frac{3x^4}{x} = 3x^3$

Exponent Name Key ID: 1

Multiplying Powers

Simplify. Your answer should contain only positive exponents.

1) $5v^2 \cdot 6v^4$

$30v^6$

2) $7k^2 \cdot 7k^3$

$49k^5$

3) $8n^3 \cdot 3n^2$

$24n^5$

4) $n^4 \cdot 6n^1$

$6n^5$

5) $x^4 \cdot 3x$

$3x^5$

6) $5n \cdot 8n^3$

$40n^4$

7) $3v^4 \cdot 3v^2$

$9v^6$

8) $5x^4 \cdot 4x^2$

$20x^6$

9) $7a \cdot 5a^3$

$35a^4$

10) $5n^1 \cdot n^3 \cdot 4n^4$

$20n^8$

11) $8v^3 \cdot 4v$

$32v^4$

12) $6p^3 \cdot 8p^2$

$48p^5$

13) $5m \cdot 3m$

$15m^2$

14) $6a \cdot 5a^4$

$30a^5$

15) $8m^3 \cdot m^4 \cdot 2m^3$

$16m^{10}$

16) $6a \cdot 7a$

$42a^2$

17) $7v^2 \cdot 2v^4 \cdot 6v^2$

$84v^8$

18) $7x^2 \cdot 3x^4$

$21x^6$

19) $8x^2 \cdot 8x^3$

$64x^5$

20) $7p \cdot 5p^2$

$35p^3$

21) $7n^2 \cdot 5n^4$

$35n^6$

22) $n^4 \cdot 7n^4$

$7n^8$

23) $3x^3 \cdot x$

$3x^4$

24) $7x \cdot 5x$

$35x^2$

25) $8a^2 \cdot 4a^3$

$32a^5$

26) $8x^4 \cdot 4x^4$

$32x^8$

27) $3x \cdot 6x$

$18x^2$

28) $3x^4 \cdot 7x^4$

$21x^8$

29) $6x^4 \cdot 2x$

$12x^5$

30) $2n^2 \cdot 6n^4 \cdot n^3$

$12n^9$

Math 306: Power of a Power $(x^m)^n = x^{mxn}$ Name _____ ID: 1

Simplify. Your answer should contain only positive exponents.

1) $(5x^3)^2$

$[(5x^3)(5x^3)]$

$5^2 x^{3\times 2} \rightarrow 25x^6$

2) $(3n^4)^2$

$9n^8$

3) $(5v^4)^3$

$5^3 v^{12}$ or $125v^{12}$

4) $(6a^4)^2$

$36a^8$ or $6^2 a^8$

5) $(2x^4)^3$

$2^3 x^{12}$ or $8x^{12}$

6) $(7k^3)^3$

$7^3 k^9 \rightarrow 343k^9$

7) $(5a^2)^4$

$5^4 a^8$ or $625a^8$

8) $(4v)^3$

$4^3 v^3$ or $64v^3$

9) $(6x)^3$

$6^3 x^3$ or $216x^3$

10) $(2x)^4$

$2^4 x^4$ or $16x^4$

11) $(5n^4)^2$

$5^2 n^8$ or $25n^8$

12) $(5n^2)^2$

$5^2 n^4$ or $25n^4$

13) $(4r^3)^2$

$4^2 r^6$ or $16r^6$

14) $(2v^2)^3$

$2^3 v^6$ or $8v^6$

15) $(7n^3)^4$

$7^4 n^{12}$

$2401n^{12}$

16) $(7x^2)^2$

$7^2 x^4$

$49x^4$

17) $(6m^3)^3$

$6^3 m^9$

$216m^9$

18) $(8r^3)^2$

$8^2 r^6$

$64r^6$

19) $(5x^2)^3$

$5^3 x^6$

$125x^6$

20) $(a^3)^4$

a^{12}

21) $(m^3)^3$

m^9

22) $(6n)^2$

$6^2 n^2$

$36n^2$

23) $(3x^4)^4$

$3^4 x^{16}$

$81x^{16}$

24) $(6r^2)^4$

$6^4 r^8$

$1296r^8$

25) $(3p)^4$

$3^4 p^4$

$81p^4$

26) $(7k^2)^3$

$7^3 k^6$

$343k^6$

27) $(5n^3)^3$

$5^3 n^9$

$125n^9$

28) $(2x^3)^3$

$2^3 x^9$

$8x^9$

29) $(8m^4)^3$

$8^3 m^{12}$

$512m^{12}$

30) $(3x^2)^2$

$3^2 x^4$

$9x^4$